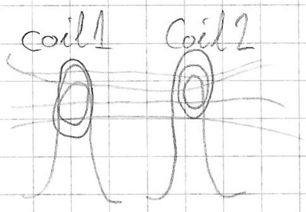
Inductance, Electromagnetic Oscillations and AC circuits

- A changing magnetic flux through a circuit induces EMF
- An electric current produces a magnetic field

→ Changing current in a circuit induce a current and EMF in another circuit

→ even an EMF in itself

Mutual inductance



If two coils of wire are placed in close proximity, a changing current in one will induce a current in the other

ϕ_{21} : Magnetic flux through each loop in coil 2 due to current in coil 1

$N_2\phi_{21}$: total magnetic flux in coil 2 due to coil 1

$$N_2\phi_{21} \propto I_1$$

$$M_{21} = \frac{N_2\phi_{21}}{I_1}$$

→ proportionality constant is called "mutual inductance"

$$\mathcal{E}_2 = -N_2 \frac{d\theta_{21}}{dt} \quad \text{(Faraday's Law)}$$

$$= -N_2 \frac{d}{dt}\left(\frac{M_{21} I_1}{N_2}\right) = -M_{21}\frac{dI_1}{dt}$$

$$\boxed{\mathcal{E}_2 = -M_{21}\frac{dI_1}{dt}}$$

Mutual inductance depends only on geometric factors such as size, shape, number of turns relative positions

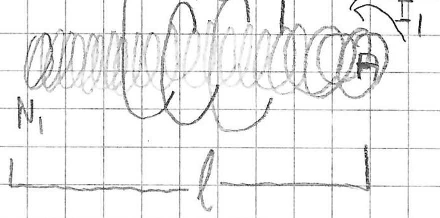
Conservation of energy $\rightarrow M_{21} = M_{12}$

$$\mathcal{E}_1 = -M\frac{dI_2}{dt}$$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt}$$

The SI unit for mutual inductance is henry (H), where $1H = 1V.s/A = 1\Omega.s$

Example Solenoid and Coil



A long thin solenoid of length l and cross sectional area A

Assume all the flux from coil 1 passes through coil 2

$$\phi_{21} = BA = \mu_0 \frac{N_1}{l} I_1 A$$

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$$M = \frac{N_2 \phi_{21}}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

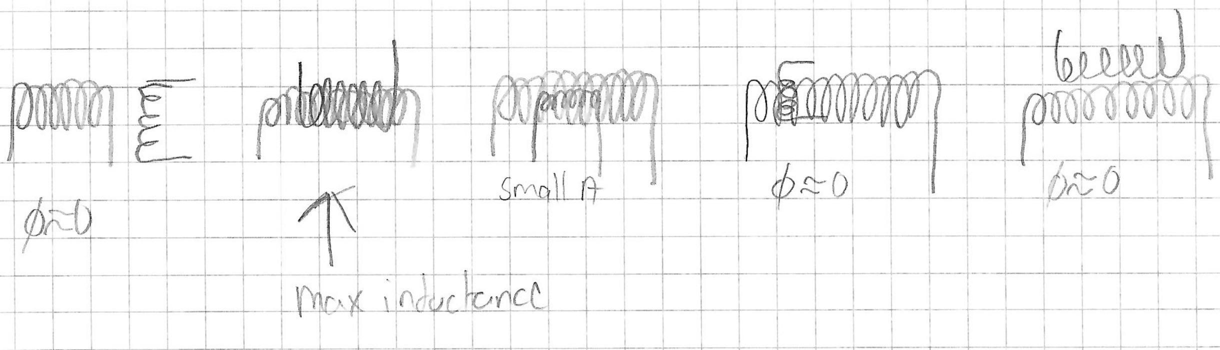
fringe fields

- M_{21} is easy to calculate, M_{12} is not since $M_{12} = M_{21}$ we use the simpler one

- If the second coil is inside the first A would refer to the area of second solenoid

- A transformer is a device which the coupling is maximised

- Mutual inductance is not always desirable, especially in electronic circuits which has fast switching, low current elements such as modern cpu's



Self - Inductance

Lenz's law: When you try to change the current through the coil, an emf that opposes the change is induced

Increasing I → increasing magnetic flux
→ opposing induced flux
decreasing I → decreasing magnetic flux
→ fortifying induced flux

The magnetic flux ϕ_B passing through the N turns of coil is proportional to the current I passing the coil

→ we define self inductance L as

$$L = \frac{N\phi_B}{I}$$

The induced emf in the coil is, from Faraday's law

$$\mathcal{E} = -N\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$$

Like mutual inductance, self inductance is measured in Henrys

The magnitude of L depends on geometry and materials such as ferromagnetic core

Self inductance ≡ inductance

There is always some inductance in a circuit albeit small

Significant inductance → inductor

resistance of the inductor has to be handled seperately

 = realistic inductor

the inductance of resistors are minimized by noninductive winding (winding back onto itself to cancel out ϕ)

$$\mathcal{E} = -L\frac{dI}{dt}$$

→ if ac current is fed this element will "resist" the current flow

Larger the L, Larger the resistance (later we will also see that this also depends on the frequency of the ac current)

This is called reactance or impedance of the inductor

a DC current through an inductor will likely burn it out, since they usually have very small internal resistance

Common inductors: $1\mu H \rightarrow 1H$

Solenoid inductance

a) Determine a formula for the self inductance L of a tightly wrapped and long solenoid containing N turns of wire, in length l, with cross section area A

$B = \mu_0 \frac{N}{l} I$ (ignoring the fringes)

$$\phi_B = BA = \frac{\mu_0 N I A}{l}$$

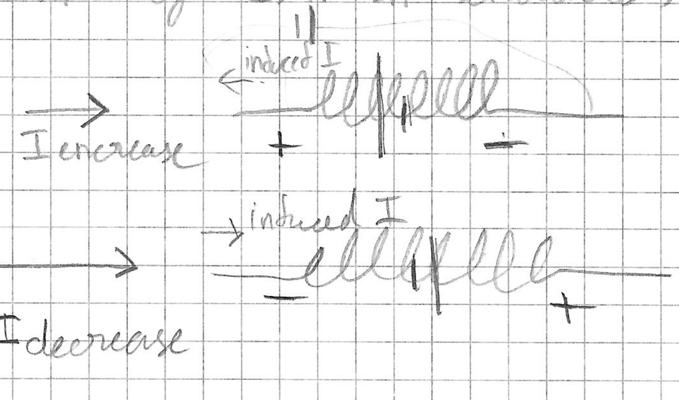
$$L = \frac{N\phi_B}{I} = \frac{\mu_0 N^2 A}{l}$$

b) $N = 100$, $l = 5.0\,cm$, $A = 0.30\,cm^2$ (air filled)

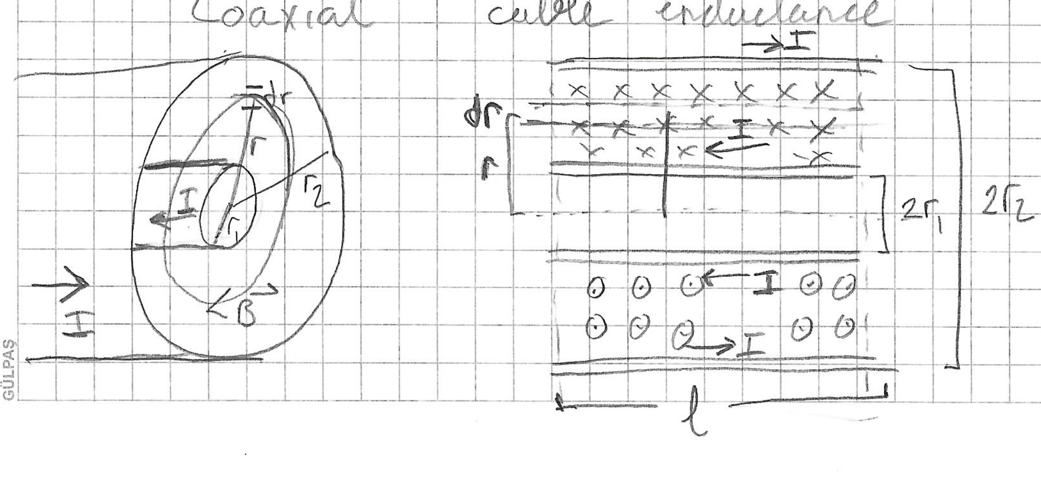
$$L = \frac{(4\pi \times 10^{-7}\ T.m/A)(100)^2(3.0\times 10^{-5}\,m^2)}{(5.0 \times 10^{-2}\,m)} = 7.5\,\mu H$$

(note: for an air filled solenoid, stray fields are significant → this is just an approximation)

Direction of EMF in inductor



Coaxial cable inductance



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Determine the inductance per unit length of a coaxial cable. Assume the conductors are thin hollow tubes. Magnetic field inside both these conductors can be ignored

Ampère's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$\oint$ = circle at a distance r from the center

$$B = \frac{\mu_0 I}{2\pi r}$$

$$d\phi_B = B\underbrace{(l\,dr)}_{dA} = \frac{\mu_0 I l}{2\pi r}\, l\,dr$$

$$\phi_B = \int d\phi_B = \frac{\mu_0 I l}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$$

$N = 1$

$$L = \frac{\phi_B}{I} = \frac{\mu_0 l}{2\pi} \ln \frac{r_2}{r_1}$$

inductance per unit length is

$$\frac{L}{l} = \frac{\mu_0}{2\pi} \ln \frac{r_2}{r_1}$$

(note this does not depend on I)

Energy stored in a magnetic field

Energy Stored in a magnetic field

L carrying I with $\frac{dI}{dt}$ rate of change

$$P = I\mathcal{E} = LI\frac{dI}{dt}$$ (no - because we drive the solenoid internally)

$$dW = P\,dt = LI\,dI$$

$$W = \int dW = \int_0^I LI\,dI = \frac{1}{2}LI^2$$

(work - energy)

$$U = \frac{1}{2}LI^2$$ (similar to $U = \frac{1}{2}CV^2$)

solenoid:

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al$$

(this energy can be thought to reside in the volume enclosed by the windings

$$\text{energy density} = u = \frac{1}{2}\frac{B^2}{\mu_0}$$

This formula is valid in general!

$\mu_0 \to \mu$ for magnetic materials

(similar to $\frac{1}{2}\epsilon_0 E^2$)

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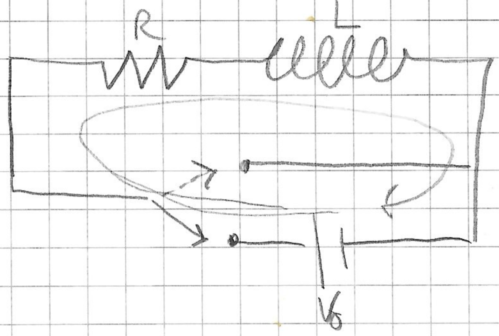
LR circuits



Kirchoff's at the instant switch is connected to battery

$$V_0 - IR - \underbrace{L\frac{dI}{dt}}_{\varepsilon} = 0$$

$$L\frac{dI}{dt} + RI = V_0$$

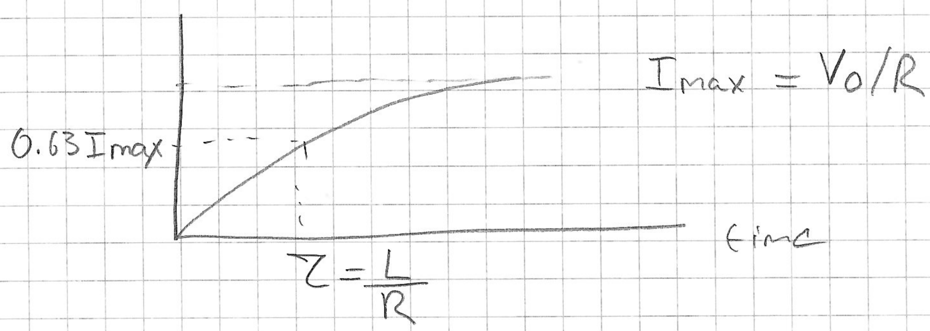
$$\int_{I=0}^{I} \frac{dI}{V_0 - IR} = \int_0^t \frac{dt}{L}$$

$$-\frac{1}{R}\ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{t}{L}$$

$$I = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R} \quad \text{(time constant for LR circuit)}$$

I reaches $(1 - 1/e) = 0.68$ after τ



When the switch is in discharge position

$$L\frac{dI}{dt} + RI = 0$$

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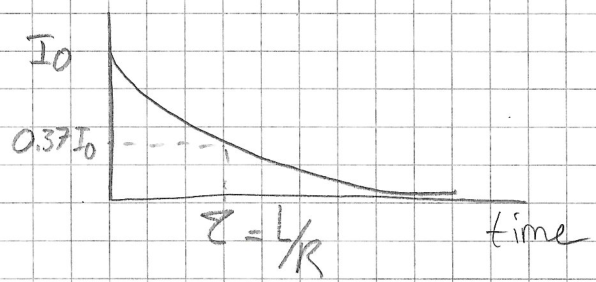
$$\int_{I_0}^{I} \frac{dI}{I} = -\int_0^t \frac{R}{L}\,dt$$

$$\ln \frac{I}{I_0} = -\frac{R}{L}t$$

$$I = I_0\, e^{-t/\tau}$$

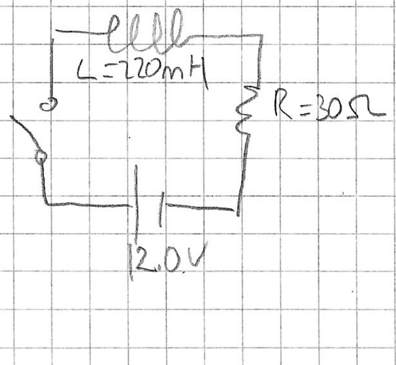


there is a reaction time when an electromagnet is turned on or off

LR circuit has similar behaviour to RC but the time constant is inversely proportional to R

Example

At $t=0$ a 12.0 V battery is connected in series with a 220 mH inductor and a total of 30 Ω resistance



a) current at $t=0$?

(inductor opposses the change so at $t=0$ $I=0$

b) what is the τ

$$\tau = L/R = 0.22\,H / 30\,\Omega = 7.3\text{ ms}$$

(c) What is the max current

$$I_{max} = V_0/R = 12.0V/30\Omega = 0.40A$$

d) How long will it take the current to reach half its maximum value

$$I = \frac{1}{2} I_{max} = \frac{V_0}{2R}$$

$$1 - e^{-t/\tau} = \frac{1}{2}$$

$$e^{-t/\tau} = \frac{1}{2}$$

$$t = \tau \ln 2 = (7.3 \times 10^{-3} s)(0.69) = 5.0 ms$$

e) At this instant, at what rate is energy being delivered by the battery?

$$I = I_{max}/2 = 200 mA$$

$$P = IV = (0.20A)(12V) = 2.4W$$

f) at what rate inductor stores energy?

$$U = \frac{1}{2} L I^2$$

$$\frac{dU}{dt} = LI\frac{dI}{dt} = I\left(L\frac{dI}{dt}\right) = I(V_0 - RI)$$

$$= 2.4W - \underbrace{(30\Omega)(0.20A)^2}_{\text{lost to heat}} = 1.2W$$

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