Energy is a very subtle concept. It is very difficult to get it right
—

Conservation means this: There is a number, which you can calculate, at one moment, and as nature undergoes its multitude of changes, this number does not change. That is, if you calculate again, this quantity, it'll be the same as before. An example is the conservation of energy: There's a quantity that you can calculate according to a certain rule, and it comes out the same answer no matter what happens.

—Feynman

This a very useful thing! Using this we can deal with situations where applying Newton's laws is difficult or impossible

Conservative and Non-conservative forces:

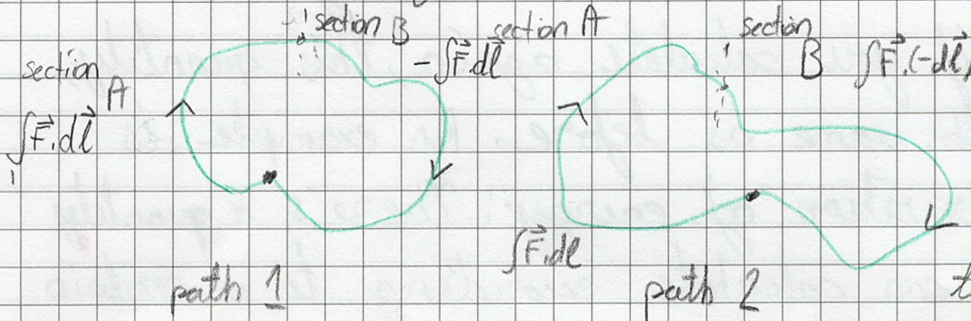
Conservative force (I)

the work done by the force on an object moving from one point to another depends only on the initial and final positions of the object, and is independent of the path taken

→ function only of position (no time, velocity etc.)

Conservative force (II)

a force is conservative if the net work done by the force on an object moving around any closed path is zero



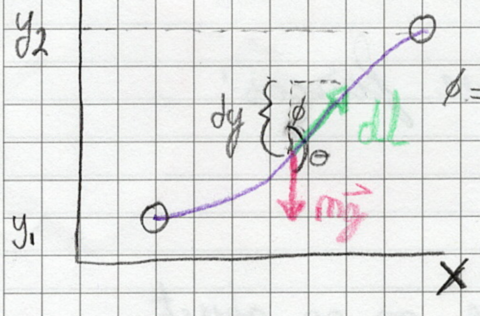
if the force is conservative, both sections have same amount of work required

thus $\Sigma W = 0$

→ the work done by a conservative force is recoverable

ex. gravity

$$W_G = \int_1^2 \vec{F}_G \cdot d\vec{l} = \int_1^2 mg \cos\theta \, dl = -\int_{y_1}^{y_2} mg \, dy$$



$\phi = 180° - \theta$ $(\cos\theta = -\cos\phi)$

$dy = dl \cos\phi$

$$= -mg(\underbrace{y_2 - y_1}_{h})$$

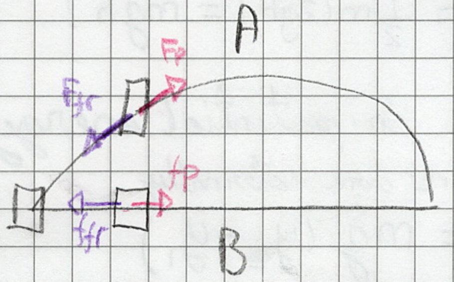
⇒ does not depend on path!

Chapter 8 P2 V1

....../....../.......

Conservative forces	Non Conservative forces
Gravitational	Friction
Elastic	Air resistance
Electric	Tension in cord
	Motor or rocket propolsion
	push or pull by a person

i.e.



due to friction, path A needs more work, both form (I) and (II) are not satisfied

Potential energy

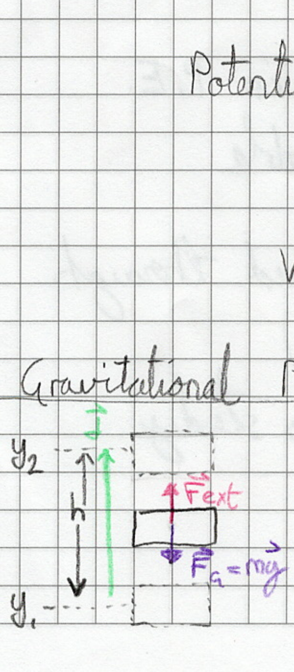
Kinetic energy : energy associated with a moving object

$$K = \frac{1}{2}mv^2$$

Potential energy : stored energy that has potential to do work

Various forces → various forms of potential energy

Gravitational Potential energy



if someone is raising a block from y_1 to y_2 at constant speed, this person is supplying an "external" force. This external force does

$$W_{ext} = \vec{F}_{ext} \cdot \vec{d} = mgh\cos 0° = mgh = mg(y_2 - y_1)$$

the work done by the gravitational force is

$$W_G = \vec{F}_G \cdot \vec{d} = mgh\cos 180° = -mgh = -mg(y_2 - y_1)$$

opposite direction!

notice, if the person let's the object go at y_2 after free falling to y_1, the object would gain $v^2 = 2gh$, thus at y_1, it would have a kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$!

potential energy: U change in potential energy: ΔU

work done externally

$$\Delta U = U_2 - U_1 = W_{ext} = mg(y_2 - y_1)$$

or

potential of the force to do work

$$\Delta U = U_2 - U_1 = -W_G = mg(y_2 - y_1)$$

since g contains both Earth and the mass, this is the potential energy of the Earth-mass system

What physically matters: ΔU the change in P.E.

ΔU is related to work done → is measurable

Potential energy belongs to a system. (defined through force which always requires something else)

System: One or more objects we choose to study

Ex

goes to 2 then 3

a) gravitational potential energy at points 2 and 3 relative to point 1

b) What is the change in potential energy when the car moves from point 2 to 3?

c) Calculate everything relative to 3

a) $y_1 = 0$ $\quad y_2 = +10\text{ m}$ $\quad U_2 = mgy_2 = (1000\text{kg})(9.8\text{m/s}^2)(10\text{m}) = 9.8\times10^4\text{ J}$

$y_3 = -15\text{m}$ $\quad U_3 = mgy_3 = (1000\text{ kg})(9.8\text{ m/s}^2)(-15\text{m}) = -1.5\times10^5\text{ J}$

b) $\Delta U = U_{final} - U_{initial} = U_3 - U_2 = (-1.5\times10^5\text{ J}) - (9.8\times10^4\text{ J})$

$= -2.5\times10^5\text{ J}$

c) $y_3 = 0$ $\quad y_1 = +15\text{m}$ $\quad y_2 = +25\text{m}$

$U_1 = (1000\text{ kg})(9.8\text{ m/s}^2)(15\text{m}) = 1.5\times10^5\text{ J}$

$U_2 = 2.5\times10^5\text{ J}$

$U_3 = 0 \quad (y_3 = 0)$

change in P.E. going from 2 to 3

$U_3 - U_2 = 0 - 2.5\times10^5\text{ J} = -2.5\times10^5\text{ J}$

(the same as above)

note Work done by gravity depends only on height, not the path taken

Potential energy in general

The concept of potential energy is only valid for conservative forces. (Path independence is mandatory)

$$\Delta U = -W_g = -\int_1^2 \vec{F}_g \cdot d\vec{l}$$

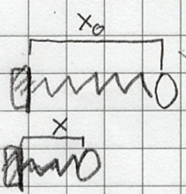
$\downarrow$ for other forces

$$\boxed{\Delta U = U_2 - U_1 = -\int_1^2 \vec{F} \cdot d\vec{l} = -W}$$

(remember, the external force supplies this, thus, it is negative of above)

Elastic potential energy

Elastic material model: Spring



$\to$ zero P.E.

$\hookrightarrow$ energy that can be converted to K.E.

$$F_s = -k\Delta x = -kx$$

$\nearrow$ compression

$\searrow$ spring constant

$$\Delta U = U(x) - U(0)$$

$$= -\int_1^2 \vec{F}_s \cdot d\vec{l} = -\int_0^x (-kx)\,dx = \frac{1}{2}kx^2$$

$$\to \boxed{U_{el} = \frac{1}{2}kx^2}$$

1-D conservative forces

$$\boxed{U(x) = -\int F(x)\,dx + C}$$

— U at $x=0$ (set $C=0$)

$$\frac{d}{dx}\int F(x)\,dx = F(x) \to \boxed{F(x) = -\frac{dU(x)}{dx}}$$

Example Determine F from U.

Suppose $U(x) = -ax/(b^2+x^2)$ (a, b : constants)

$F(x) = ?$

$$F(x) = -\frac{dU}{dx} = -\frac{d}{dx}\left[-\frac{ax}{b^2+x^2}\right] = \frac{a}{b^2+x^2} - \frac{ax}{(b^2+x^2)^2}2x$$

$$= \frac{a(b^2-x^2)}{(b^2+x^2)^2}$$

3D:

$$F_x = -\frac{\partial U}{\partial x}, \quad F_y = -\frac{\partial U}{\partial y}, \quad F_z = -\frac{\partial U}{\partial z}$$

$$\vec{F}(x,y,z) = -\hat{i}\frac{\partial U}{\partial x} - \hat{j}\frac{\partial U}{\partial y} - \hat{k}\frac{\partial U}{\partial z}$$

Mechanical Energy and its Conservation

Conservative system (only conservative forces do work)

→ we can define potential energy

$W_{net} = \Delta K$ (work-Energy principle)

$\Delta U_{total} = -\int_1^2 \vec{F}_{net} \cdot d\vec{l} = -W_{net}$

$\boxed{\Delta K + \Delta U = 0}$ conservative forces only!

$(K_2 - K_1) + (U_2 - U_1) = 0$ (conservative forces only!)

$$\underbrace{K_2 + U_2}_{E_2} = \underbrace{K_1 + U_1}_{E_1} = \text{constant}$$

$E = K + U$; "Energy"

principle of conversation of mechanical energy:

If only conservative forces are doing work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant, it is conserved.

conservative force → mechanical energy is conserved

Problem solving using conservation of mechanical E

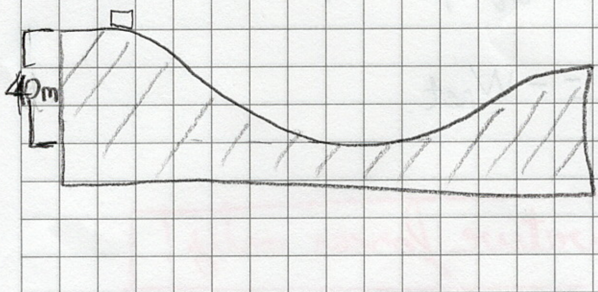
$y_1 = 3.00\,m$ what is the speed of the rock at y_2?

$$E = K + U = \frac{1}{2}mv^2 + mgy$$

$y_2 = 1.00\,m$ all conservative forces: $E_1 = E_2$

$$\frac{1}{2}\cancel{m}\cancel{v_1^2}^{\,0} + \cancel{m}gy_1 = \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}gy_2$$

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.8\,m/s^2)[(3.0\,m) - (1.0\,m)]}$$
$$= 6.3\,m/s$$



a) Speed of the roller coaster at the bottom

b) at what height it will have half this speed?

a) $\frac{1}{2}\cancel{m}\cancel{v_1^2}^{\,0} + \cancel{m}gy_1 = \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}gy_2 \rightarrow v_2 = \sqrt{2gy_1} = \sqrt{2(9.8\,m/s^2)(40\,m)}$

$= 28\,m/s$

b) $y_2 = y_1 - \frac{v_2^2}{2g} = 40\,m - \frac{(14\,m/s)^2}{2(9.8\,m/s^2)} = 30\,m$

Chapter 8 PS V1

A suggestion
- If the forces involved are constant both Newton's laws approach and work-energy approach is feasible
- If the forces are -not- constant, and/or the path is not simple: work-energy is probably better

Ex: Toy dart gun

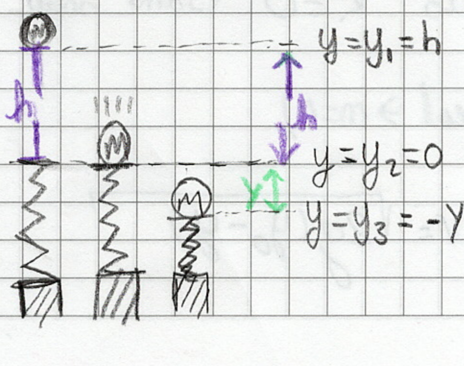
A dart of mass 0.100 kg is pressed against the spring of a toy dart gun. The spring has a stiffness constant $k = 250$ N/m and ignorable mass. It is compressed 6.0 cm. If the dart releases at 0. cm, which is the natural length of the spring, what is the speed of the dart?

$K_1 = 0$ (dart is initially at rest)

$$0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + 0 \rightarrow v_2 = \sqrt{\frac{(250\ N/m)(-0.060m)^2}{(0.100\ kg)}} = 3.0\ m/s$$

Ex: Two types of potential energy

A ball of mass $m = 2.60$ kg, starting from the rest, falls a vertical distance $h = 55.0$ cm before striking a vertical coiled spring, which it compresses an amount $Y = 15$ cm. Determine the spring constant, ignore mass of the spring and air resistance.



$$\frac{1}{2}m\cancel{v_1^2}^{\,0} + mg\,y_1^{\,h} = \frac{1}{2}mv_2^2 + \cancel{mgy_2}^{\,0}$$

$$mgh = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gh} = \sqrt{2(0.55\,m)(9.8\,m/s^2)} = 3.28\ m/s$$

$$\frac{1}{2}mv_2^2 + mg\cancel{y_2}^{0} + \frac{1}{2}\cancel{ky_2^2}^{0} = \frac{1}{2}\cancel{mv_3^2}^{0} + mgy_3 + \frac{1}{2}ky_3^2$$

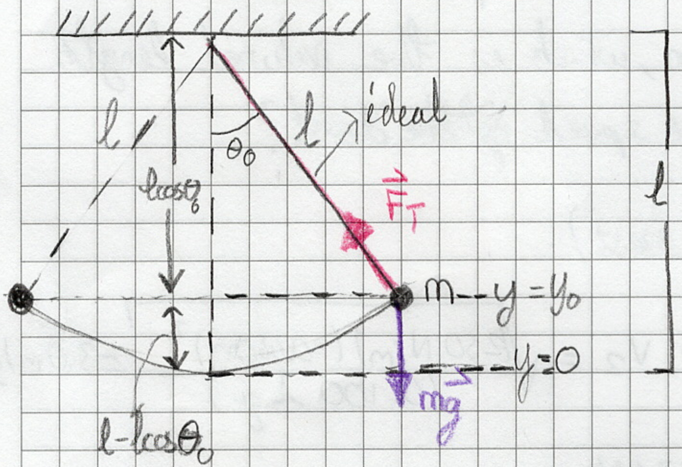
$$-mgY + \frac{1}{2}k(-Y)^2 = 0$$

$$k = \frac{2}{Y^2}\left[\frac{1}{2}mv_2^2 + mgY\right] = \frac{m}{Y^2}\left(v_2^2 + 2gY\right)$$

$$= \frac{(2.60\,kg)}{(0.150\,m)^2}\left[(3.283\,m/s)^2 + 2(9.80\,m/s^2)(0.150\,m)\right]$$

$$= 1590\ N/m$$

Ex: Swinging pendulum



at $t=0$ pendulum is released from θ_0 to the vertical

a) describe the motion using kinetic energy and potential energy.

b) Speed of the bob as a function of θ

c) lowest point in the spring

d) Tension in the cord

a) at start $K=0$. At lowest point $K=max$ $U=min$

continues the motion until $U=U_0$ $K=0$ (same angle)

b) $E_{initial} = \frac{1}{2}\cancel{mv^2}^{0} + mgy_0$ (cord is ideal $\rightarrow m=0$)

$$E = \frac{1}{2}mv^2 + mgy = E_{initial} = mgy_0 \rightarrow v = \sqrt{2g(y_0 - y)}$$

$$(y_0 - y) = (l - l\cos\theta_0) - (l - l\cos\theta) = l(\cos\theta - \cos\theta_0)$$

$$\rightarrow v = \sqrt{2gl(\cos\theta - \cos\theta_0)}$$

c) at lowest point $y=0$

$$\rightarrow v = \sqrt{2gy_0} = \sqrt{2gl(1-\cos\theta_0)}$$

d) tension does not do work in the system

$$m\frac{v^2}{l} = F_T - mg\cos\theta \rightarrow F_T = m\left(\frac{v^2}{l} + g\cos\theta\right) = 2mg(\cos\theta - \cos\theta_0) + mg\cos\theta$$

$$F_T = (3\cos\theta - 2\cos\theta_0)mg$$

The Law of conservation of Energy

In common natural processes, the mechanical energy decreases. Some processes dissipate the mechanical energy into other forms. The result of these processes is called "dissipative force" i.e. friction thermalises the mechanical energy (converts it into thermal energy)

The total energy is mechanical energy + all other forms of energy

As far as we know emprically

$$\boxed{\Delta K + \Delta U + [\text{change in all other forms of energy}] = 0}$$

Law of conservation of energy

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant.

one of the most important principles of physics!

Energy Conservation with dissipative forces

From the work-energy principle, the net work W_{net} done on an object is equal to the change in its kinetic energy:

$$\Delta K = W_{net}$$

$$W_{net} = \underbrace{W_C}_{\text{conservative}} + \underbrace{W_{NC}}_{\text{non-conservative}}$$

$$W_C = \int_1^2 \vec{F} \cdot d\vec{l} = -\Delta U$$

thus

$$\Delta K = W_C + W_{NC} = -\Delta U + W_{NC}$$

hence

$$\boxed{(\Delta K + \Delta U) = W_{NC}}$$

i.e., a non conservative force changes the mechanical energy.

i.e. friction

$$W_{NC} = -F_{fr}\,l = \Delta K + \Delta U = \left(\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2\right) + (mgy_2 - mgy_1)$$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 + F_{fr}\,l$$

change in mechanical energy

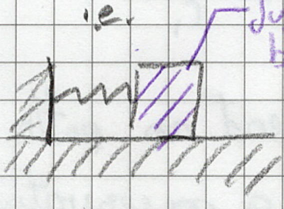
Work-Energy vs Energy Conservation

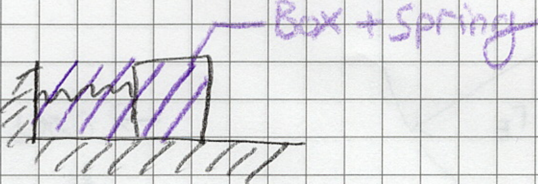
Work-Energy should not be viewed as a statement of conservation of energy.

→ in the presence of external forces
Work-Energy

→ no external forces
conservation of energy

i.e.





Work-energy
(spring exerts an external force to system)

Conservation of Energy
(no external forces act on system)

Example: Friction with a spring

our system does not have external forces

$$E_{initial} = E_{final}$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}kX^2 + \mu_k mgX$$

$$\mu_k = \frac{v_0^2}{2gX} - \frac{kX}{2mg}$$

Gravitational potential energy and escape velocity

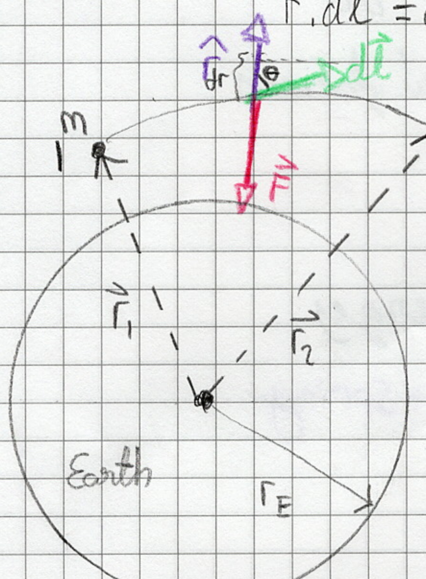
as we move further from the planet, we can not substitute F_G with mg anymore

$$\vec{F} = -G \frac{m M_E}{r^2} \hat{r} \qquad [r > r_E]$$

Work done moving from a point to another under the gravity amounts to

$$W = \int_1^2 \vec{F} \cdot d\vec{l} = -G m M_E \int_1^2 \frac{\hat{r} \cdot d\vec{l}}{r^2}$$

$$\hat{r} \cdot d\vec{l} = dr \quad (d\vec{l} \text{ along } \hat{r})$$



$$W = -G m M_E \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= G m M_E \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$W = \frac{G m M_E}{r_2} - \frac{G m M_E}{r_1}$$

→ does not depend on path thus it is a conservative force : Potential energy!

$$\Delta U = U_2 - U_1 = -\frac{G m M_E}{r_2} + \frac{G m M_E}{r_1}$$

$$\boxed{U(r) = -\frac{G m M_E}{r} \qquad r > r_E}$$

$$\boxed{\frac{1}{2} m v_1^2 - \frac{G m M_E}{r_1} = \frac{1}{2} m v_2^2 - \frac{G m M_E}{r_2}}$$ gravity only

Escape velocity

unless you surpass "the escape velocity", you always come back to earth

$$\frac{1}{2}mv_{ex}^2 - \frac{GmM_E}{r_E} = \underbrace{0 + 0}_{r \to \infty}$$

particle is just free

$$v_{esc} = \sqrt{2GM_E/r_E} = 1.12 \times 10^4 \, m/s$$

Power

Power is the rate at which work is done

The average power is $\bar{P}$

$$\boxed{\bar{P} = \frac{W}{t} = \frac{\text{energy transformed}}{\text{time}}}$$

The instantaneous power is P

$$\boxed{P = \frac{dW}{dt} = \frac{dE}{dt}}$$

i.e. it is the rate the energy is transformed

in SI units the unit of power is watt

$1W = 1 J/s$ (James Watt actually invented and used hp (horsepower))

it is often convenient to write power as

$$\boxed{P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{l}}{dt} = \vec{F} \cdot \vec{v}}$$

ex. Calculate the power required of a 1400 kg car under a) climbing 10' hill at 80 km/h b) accelerate from 90 to 110 km/hr @ level ground in 6.0 s assume the retardation force is $F_R = 700N$

note: retardation force is due to air. the friction force is on tires, and is necessary to accelerate the car

a) $F = 700N + mg\sin 10' = 700N + (1400\ kg)(9.80\ m/s^2)(0.174)$
$= 3100N$

$\bar{v} = 80\ km/h = 22\ m/s$ (parallel to $\vec{F}$)

$\bar{P} = F\bar{v} = (3100N)(22\ m/s) = 6.8\times10^4 W = 68\ kW = 91\ hp$

b) $\bar{a}_x = \dfrac{(30.6\ m/s) - 25.0\ m/s)}{6.0\ s} = 0.93\ m/s^2$

$ma_x = \sum F_x = F - F_R$

$F = ma_x + F_R = (1400\ kg)(0.93\ m/s^2) + 700N = 2000N$

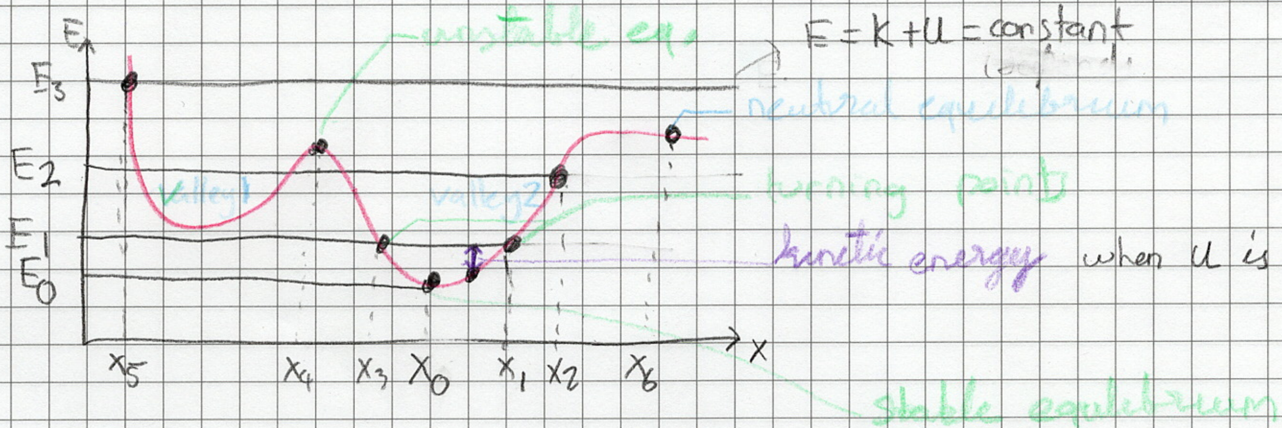
$P = \vec{F}.\vec{v} \rightarrow \bar{P} = (2000N)(30.6\ m/s) = 61.2\ kW = 82\ hp$

notice that an ICE is grossly inefficient in converting energy stored in fuel to useful work

$e = \dfrac{P_{out}}{P_{in}} \sim 15\%$ for ICEs (%85 of the energy goes warming the planet)

Potential energy diagrams

This is the graph of $U(x)$ versus x



- Kinetic energy is always >0 thus minimum of U is minimum of E
- If the system's total energy is $>E_0$ then
  $K = E - U(x)$
- When the particle has energy E_1, it oscillates between x_1 and x_3 (any value outside will make $K<0$) these are called turning points
- If the particle has E_2, it oscilates either in valley 1 or valley 2. If it has E_3 it never comes back or oscillates,
- Since $F=-dU/dx$, the curve of the slope gives the direction of the force. i.e at x_2 slope is positive force is negative (to the left)
- At $x=x_0$ slope is $0 \rightarrow F=0$ particle is at equilibrium. Looking at the second derivative x_0 is stable, x_4 is unstable, x_6 is neutral equilibrium