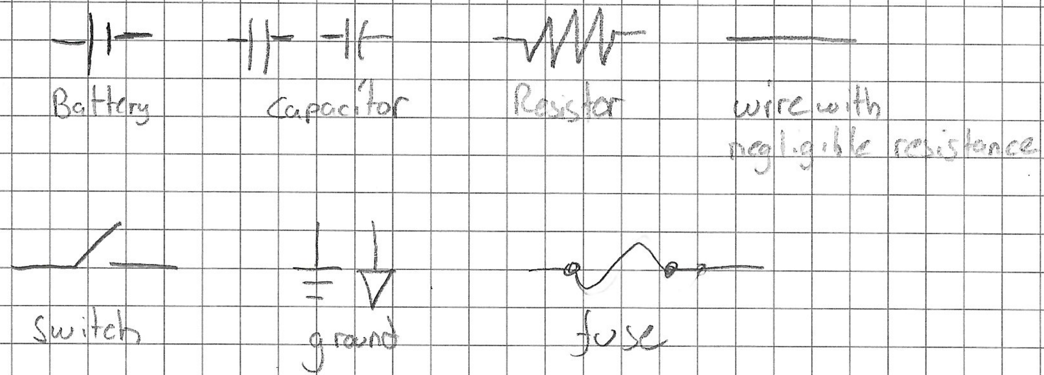
DC circuits

a "diagram" for the circuit is a graphical representation used to analyze what a circuit does



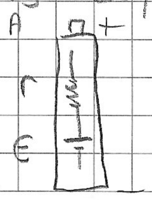
these are the circuit elements we have seen up to now

EMF and Terminal Voltage

When no current passes through the battery (the same) the potential difference between its terminals is called electromotive force (emf) ($\mathcal{E}$)
(this is not a force)

$\mathcal{E}$ is in Volts

A more realistic battery is modelled as an internal resistance, r, connected in series with perfect $\mathcal{E}$. In this manner, the voltage drop as you draw to much current from the battery is captured



$V_{AB} = \mathcal{E} - Ir$ (when discharging)

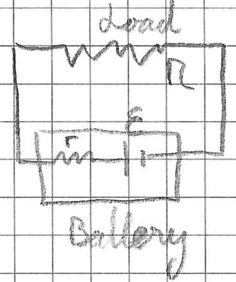
$V_{AB} = \mathcal{E} + Ir$ (when charging) example

a battery with internal resistance $r = 0.5\,\Omega$ and $12.0\,V$ is connected to a resistive load of $65\,\Omega$

a) current in the circuit?

b) the terminal voltage of the battery

c) the power dissipated by the load and the internal resistance



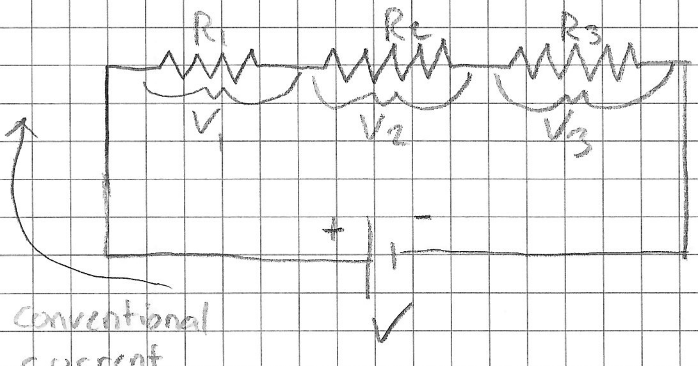
a) $$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\,V}{65.0\,\Omega + 0.5\,\Omega} = 0.183\,A$$

b) $$V_{bat} = \mathcal{E} - Ir = 12\,V - (0.183\,A)(0.5\,\Omega) = 11.9\,V$$

c) $$P_{load} = I^2 R = (0.183\,A)^2 (65)\,\Omega = 2.18\,W$$

$$P_{bat} = I^2 r = (0.183\,A)^2 (0.5)\,\Omega = 0.02\,W$$

Resistors in Series and in Parallel



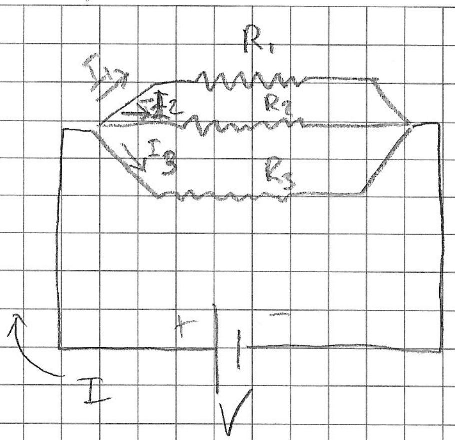
Series connection

the current through R_1 R_2 and R_3 is the same

$$V = V_1 + V_2 + V_3 = I R_1 + I R_2 + I R_3$$

$$V = I R_{eq}$$ (I is the response of the circuit to V)

$$R_{eq} = R_1 + R_2 + R_3$$



parallel connection
the potential drop along each resistance is the same

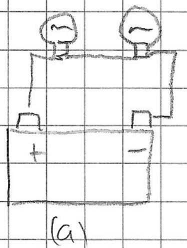
$I_1 = \frac{V}{R_1}$ $I_2 = \frac{V}{R_2}$ $I_3 = \frac{V}{R_3}$

$$I = I_1 + I_2 + I_3$$

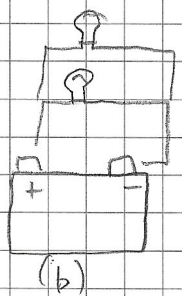
$$I = \frac{V}{R_{eq}} \rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

the net resistance is less than each single resistance

example :



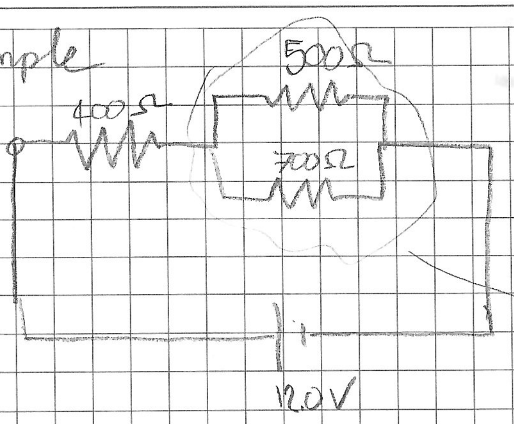
or



produces more light
(b) because in a $R_{eq} = 2R$
while in b $R_{eq} = (\frac{1}{R} + \frac{1}{R})^{-1} = \frac{R}{2}$
so $I = V/R_{eq}$ is higher in b
and $P = IV$

example
when a lamp of 60 W and 100 W are connected in parallel and in series which will be brighter
the internal resistance of 100 W lamp is lower
so in parallel it will transfer more current $P = V^2/R$
in series, The current will be the same
since 60 W bulb has higher resistance it will be brighter $P = I^2 R$

example



what is the current drawn from the battery

$$R_p = \left(\frac{1}{500\Omega} + \frac{1}{700\Omega}\right)^{-1}$$

$$= (0.002 + 0.0014)^{-1}\ \Omega$$

$$= 290\Omega$$

$$R_{eq} = 400\Omega + 290\Omega = 690\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{12.0V}{690\Omega} = 0.0174A \approx 17mA$$

example

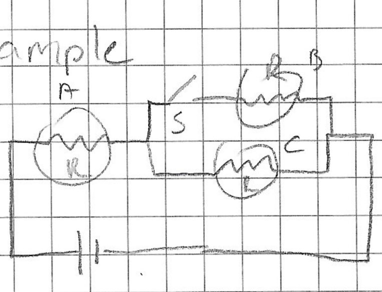
what is the current through 500 Ω above?

the potential drop across the parallel resistors is

$$V_{ab} = (0.0174\,A)(290\Omega) = 5V$$

$$I_{500\Omega} = \frac{5.0V}{500\Omega} = 10mA$$

example



—(m)— is lightbulb

compare the brightness of the bulbs when switch is closed and open

when switch is closed bulb B and C share equally the same current that passes through A, so A is the brightest

when switch is open A and C has the same current but the total resistance of the circuit is 2R instead of 1.5R so bulb C will be dimmer [illegible] above

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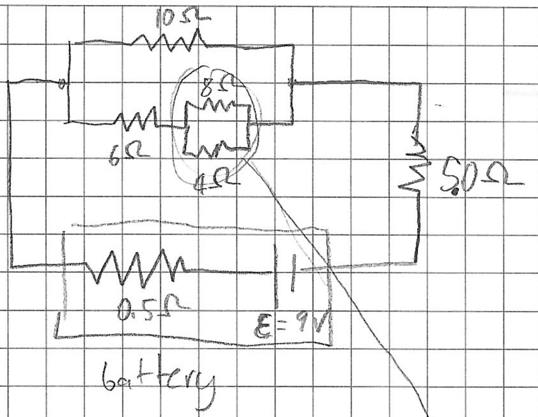
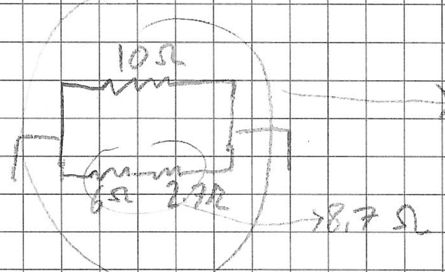


a) how much current is drawn from the battery?

b) what is the terminal voltage of the battery

c) what is the current through 6Ω resistor?

$$R_{eq_1} = \left(\frac{1}{8} + \frac{1}{4}\right)^{-1} \Omega = 2.7\Omega$$



$$R_{eq_2} = \left(\frac{1}{10} + \frac{1}{8.7}\right)^{-1} \Omega = 4.8\,\Omega$$

$$R_{eq} = \underbrace{R_{eq_2}}_{0.85\Omega} + 5.0\,\Omega + 0.5\,\Omega = 10.3\,\Omega$$

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{9.0V}{10.3\Omega} = 0.87A$$

b) $V_{ab} = \mathcal{E} - IR = 9.0V - (0.87A)(0.50\Omega) = 8.6V$

c) $4.8\,\Omega \times 0.87A = 4.176\,V$

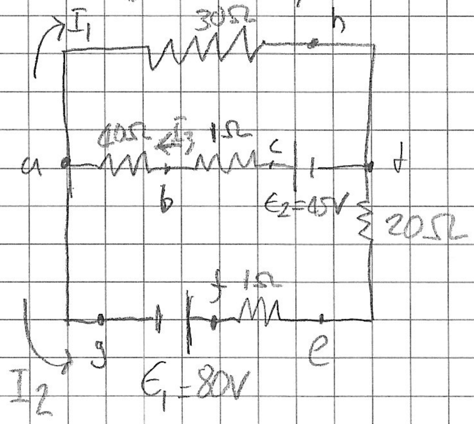
Since current through $6\,\Omega$ is the same current through 8.7Ω equivalent resistanc

$$I = \frac{4.176V}{8.7\Omega} = 0.48A$$

$V = IR$

Kirchoff's Rules

It is not always easy to "reduce" the circuit to an equivalent resistance using the techniques of the previous section



To deal with complicated circuits we use Kirchoff's rules (convenient applications of conservation of charge and energy)

(1) Junction rule

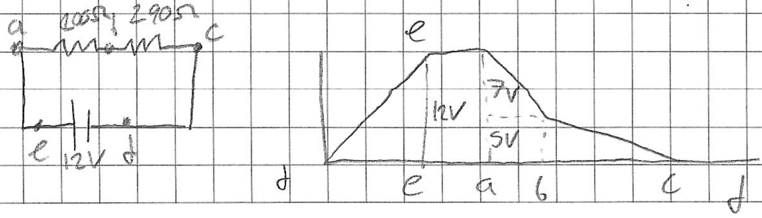
conservation of charge

at any junction point the sum of all currents entering the junction must be equal to sum of all currents leaving the junction

(2) loop rule

conservation of energy

the sum of the changes in potential around any closed loop of a circuit must be zero



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Tips

1) Label the currents

- each seperate branch should have different indices I_1, I_2, I_3
- each current refers to segment btw two junctions
- choose a direction for each current (if you have a - in the end, the actual current is in reverse direction)

2) Identify unknowns

- as many independent eqs as unknowns
- try to reduce unknowns with $V=IR$ if possible

3) Apply Kirchhoff's junction rule

4) Apply Kirchoff's loop rule

- pay attention to signs and subscripts

a) Apply Ohm's law (

- a decrease if the choosen loop direction is the same as the choosen current direction
- an increase if the choosen loop direction is apposite to choosen current direction

b) Batteries:

- if loop direction is from - to + positive
- if loop direction is from + to - negative

5) Sole

Example: solve the circuit shown in the beginning

1) Label currents
the direction of I_1 is not obvious, so we choose an arbitrary direction

2) Unknowns:
three unknowns, we need three eqs

3) Junction rule
at a $I_3 = I_1 + I_2$
at d, the same information is repeated

4) Loop rule
two loops ahdcba and ahdefga

$V_{ha} = -(I_1)(30\Omega)$ $V_{hd} = 0$ $V_{dc} = +45V$ $V_{ac} = -(I_3)(40\Omega + 1\Omega)$

$V_{ha} + V_{cd} + V_{ac} = 0 \rightarrow -30I_1 + 45 - 41I_3 = 0$

$V_{ha} = -(I_1)(30\Omega)$ $V_{dh} = 0$ $V_{ed} = +(I_2)(20\Omega)$
$V_{fe} = +I_2(1\Omega)$ $V_{gf} = -80V$ $V_{ag} = 0$

$-30I_1 + (20+1)I_2 - 80 = 0$

$I_2 = \dfrac{80 + 30I_1}{21} = 3.8 + 1.4 I_1$

$I_3 = \dfrac{45 - 30I_1}{41} = 1.1 - 0.73 I_1$

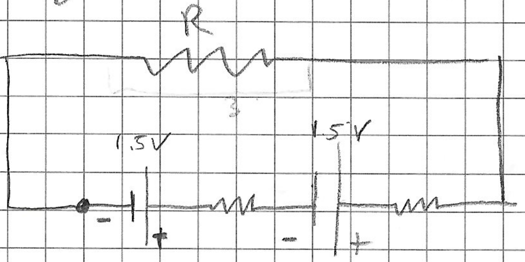
$I_1 = I_3 - I_2 = 1.1 - 0.73I_1 - 3.8 - 1.4I_1$

$3.1 I_1 = -2.7$ $I_1 = -0.87A$ (so the direction is opposite)

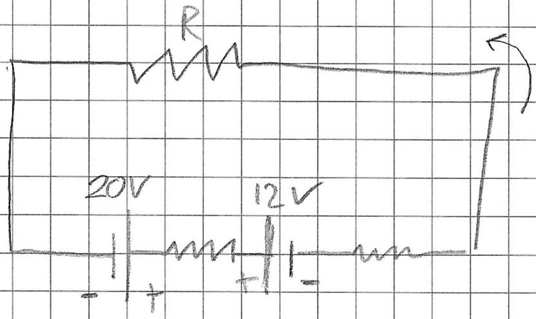
$I_2 = 3.8 + 1.4I_1 = 2.6A$ $I_3 = 1.1 - 0.73I_1 = 1.7A$

Series and Parallel EMFs
Battery charging

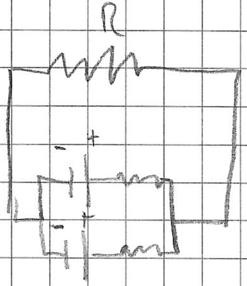
IFF the reaction in a battery is reversible, then you can use another battery (or EMF source) to charge it



series set-up for cranking up the voltage

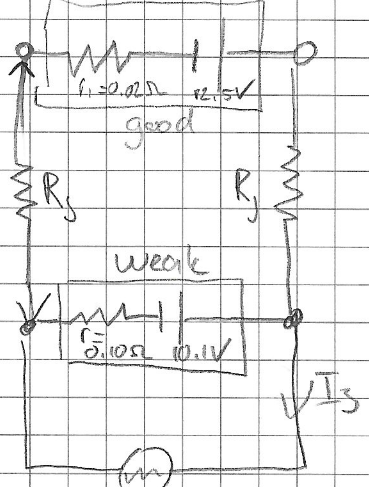


series set-up for charging



parallel set-up for for longevity (current

Jump start a car



Starter motor $\approx 0.15\Omega$

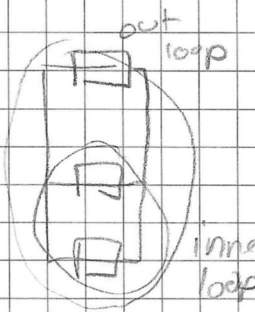
A good car battery is being used to jump start a car with weak battery using a .50 cm copper wire of 3.0 M

determine I_3 with and without the help resistivity of copper $\rho = 1.68 \times 10^{-8}\ \Omega m$

without help : $I_3 = \dfrac{10.1V}{0.25\Omega} = 40A$

with help ; $R_J = \rho l / A = (1.68\times10^{-8}\ \Omega m)(3.0m)/\pi(0.25\times10^{-2}\ m)^2$

$= 0.0026\Omega$



Loop rule

out $\left|\begin{array}{l} 12.5V - I_1(2R_J + r_1) - I_3 R_S = 0 \\ 12.5V - I_1(0.025\Omega) - I_3(0.15\Omega) = 0 \end{array}\right.$

in $\left|\ 10.1V - I_3(0.15\Omega) - I_2(0.10\Omega) = 0\right.$

$I_1 + I_2 = I_3$

$12.5V - (I_3 - I_2)(0.025\Omega) - I_3(0.15\Omega) = 0$

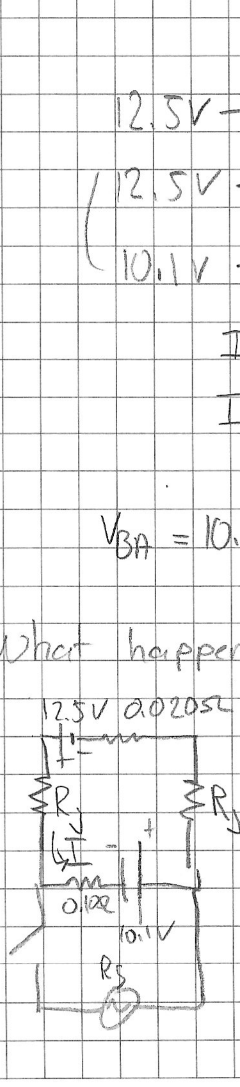
$$\left(\begin{array}{l} 12.5V - I_3(0.175\Omega) + I_2(0.025\Omega) = 0 \\ 10.1V - I_3(0.15\Omega) - I_2(0.10\Omega) = 0 \end{array}\right)$$

$I_3 = 71A$

$I_2 = -5A$ $\quad$ $I_1 = 76A$

$V_{BA} = 10.1V - (-5A)(0.10\Omega) = 10.6V$

What happens if you connect + to − ?



$12.5V + 10.1V - I(2\times R_J + 0.12\Omega) = 0$

$I = \dfrac{22.6V}{(0.005\Omega + 0.12\Omega)} \cong 180A$

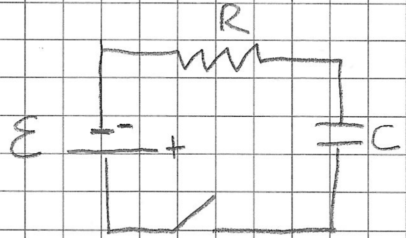
Power dissipated by weak battery

$P = I^2 R = 3200\ W$!! fire!

RC circuits

RC circuit is a circuit with a resistor and capacitor

In RC circuits we are not so interested in the final "steady state" voltage and charge on the capacitor, but rather, how these variables change in time



In this circuit, when, the switch is closed, the battery will start charging the capacitor.
as the capacitor accumulates charge the potential across its terminals will be $V_C = Q/C$

so, the loop reads: varies in time

$$\underset{\text{set}}{\mathcal{E}} = I\underset{\text{set}}{R} + Q/\underset{\text{set}}{C}$$

$I = \frac{dQ}{dt}$

$$\mathcal{E} = R\frac{dQ}{dt} + \frac{1}{C}Q$$

$$\frac{dQ}{C\mathcal{E} - Q} = \frac{dt}{RC}$$

$$\int_0^Q \frac{dQ}{C\mathcal{E} - Q} = \frac{1}{RC}\int_0^t dt$$

$$-\ln(C\mathcal{E} - Q)\Big|_0^Q = \frac{t}{RC}\Big|_0^t$$

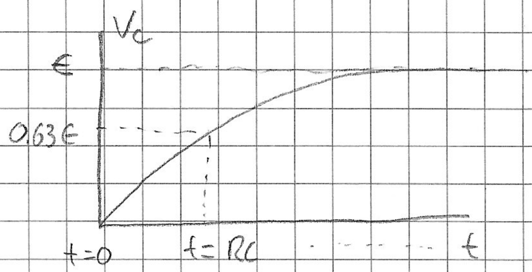
$$-\ln(C\mathcal{E}-Q) - (-\ln C\mathcal{E}) = \frac{t}{RC}$$

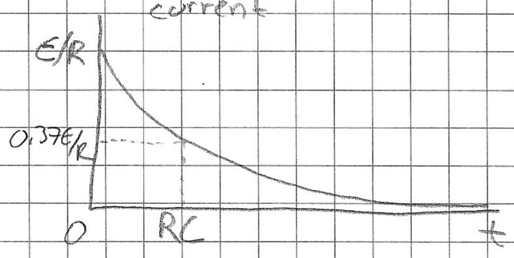
$$\ln\left(1-\frac{Q}{C\mathcal{E}}\right) = -\frac{t}{RC}$$

$$1-\frac{Q}{C\mathcal{E}} = e^{-t/RC}$$

or

$$Q = C\mathcal{E}\left(1-e^{-t/RC}\right) \rightarrow V_C = \mathcal{E}\left(1-e^{-t/RC}\right)$$





$RC = \tau$ (time constant)

τ represents the time required for the capacitor to reach $(1-e^{-1}) = 0.63$ %63 of its full charge

$$RC \rightarrow \Omega F = \frac{\cancel{V}}{A}\,\frac{C}{\cancel{V}} = s \text{ (time)}$$

$$A = \frac{\cancel{C}}{s}$$

thus RC is a measure how quickly the capacitor gets charged.

notice that the capacitor gets fully charged only when $t \to \infty$ however

2RC → %86 3RC → %95 4RC → %98

so, a lot of it gets charged pretty quickly

Chapter 26

for example $R = 20\,k\Omega$ and $C = 0.30\,\mu F$ the time constant is $(2.0\times10^{4}\,\Omega)(3.0\times10^{-7}F) = 6.0\times10^{-3}s$

$1/98 \rightarrow \frac{1}{40}\,s$

The current through the circuit is

$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

$t=0 \rightarrow I = \mathcal{E}/R$ (capacitor acts as a short circuit)

$t=\infty \rightarrow I = 0$ (capacitor acts as open switch)

Example

Simple RC circuit with $C=0.30\,\mu F$ $R=20\,k\Omega$

$\mathcal{E}=12V$

a) time constant

b) maximum charge in the capacitor

c) the time required to reach %99

d) the current when Q is half the max

e) max current

f) Charge Q when I is 0.20 of its max

a) $\tau = RC = (2.0\times10^{4}\,\Omega)(3.0\times10^{-7}\,F) = 6.0\times10^{-3}\,s$

b) $Q = C\mathcal{E} = (3.0\times10^{-7}F)(12V) = 3.6\,\mu C$

c) $Q = 0.99\,C\mathcal{E} = C\mathcal{E}(1-e^{-t/RC}) \rightarrow e^{-t/RC} = 0.01$

$$\frac{t}{RC} = -\ln(0.01) = 4.6$$

$$t = 4.6\,RC = 28\times10^{-3}\,s$$

(d) $C\mathcal{E}/2 = 1.8\,\mu C$ $\quad I = \frac{1}{R}\left(\mathcal{E} - \frac{Q}{C}\right) = \frac{1}{2.0\times10^{4}\,\Omega}\left(12V - \frac{1.8\times10^{-6}C}{0.3\times10^{-6}F}\right) = 300\,\mu A$

(e) $I_{max} = \frac{\mathcal{E}}{R} = \frac{12V}{2.0\times10^{4}\,\Omega} = 600\,\mu A$

GÜLPAŞ

(f) $Q = C(\mathcal{E} - IR) = (3.0\times10^{-7}\,F)\,[12V - (1.2\times10^{-4}A)(2.0\times10^{4}\Omega)]$

$= 2.9\,\mu C$

when discharging the capacitor

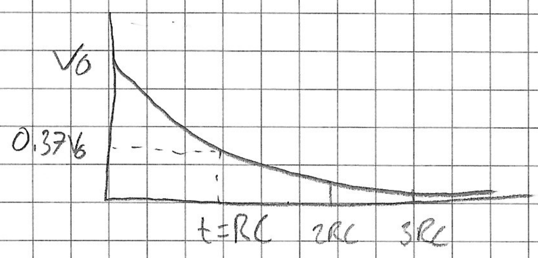
$IR = \frac{Q}{C}$

$-\frac{dQ}{dt}R = \frac{Q}{C}$

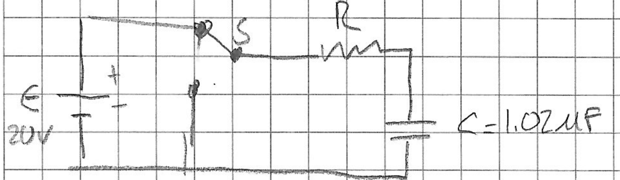
$\frac{dQ}{Q} = -\frac{dt}{RC}$

$\ln\frac{Q}{Q_0} = -\frac{t}{RC} \longrightarrow Q = Q_0 e^{-t/RC} \longrightarrow V_C = V_0 e^{-t/RC}$

$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$



example



After fully charging the capacitor, the switch is set to discharge (at $t=0$)

current decreases to %.50 after 40 µs

a) Q at $t=0$ b) $R=?$ c) Q at $t=60\,\mu s$?

$t=0$ $Q_0 = C\mathcal{E} = (1.02\times10^{-6}F)(20V) = 20.4\,\mu C$

b) $0.50\,\cancel{I_0} = \cancel{I_0}\, e^{-t/RC}$

$0.693 = t/RC$ $R = \frac{t}{0.693C} = \frac{(40\times10^{-6}s)}{0.693(1.02\times10^{-6}F)} = 57\Omega$

c) $Q = Q_0 e^{-t/RC} = (20.4\times10^{-6}C)\, e^{-\frac{60\times10^{-6}s}{57\Omega(1.02\times10^{-6}F)}} = 7.3\,\mu C$