Chapter 28 P1 V1 ....../....../.......

Magnetic field due to a straight wire

$$B \propto \frac{I}{r}$$

I → current

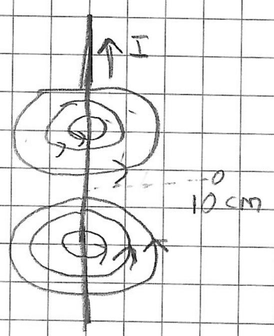
r → distance

↓

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

μ_0: permeability of free space $= 4\pi \times 10^{-7}\ T.m/A$

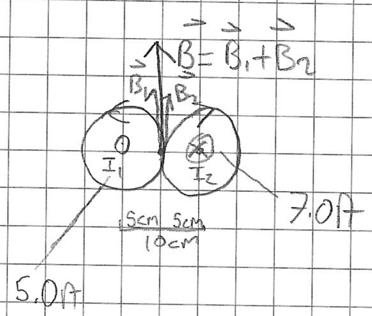
(this is chosen to be like this to make Ampere's law more elegant)



If the wire is much longer than 10 cm, a current of 25A will result in

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7}\ T.m/A\ 25A}{(2\pi)(0.10m)} = 5.0 \times 10^{-5}\ T$$

into the page due to right hand rule



$|\vec{B}| = |\vec{B}_1| + |\vec{B}_2|$ at midpoint

$$|\vec{B}_1| = B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{4\pi \times 10^{-7}\ Tm/A \times 5.0A}{2\pi\ (0.05m)}$$

$$= 2.0 \times 10^{-5}\ T$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{4\pi \times 10^{-7}\ Tm/A \times 7.0A}{2\pi\ (0.05m)} =$$

$$= 2.8 \times 10^{-5}\ T$$

$$|\vec{B}| = 4.8 \times 10^{-5}\ T$$

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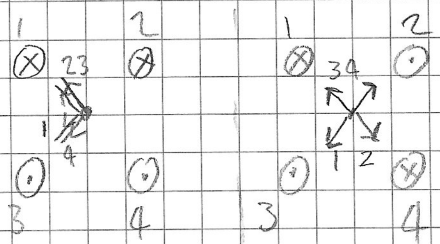
Magnetic field due to four wires,



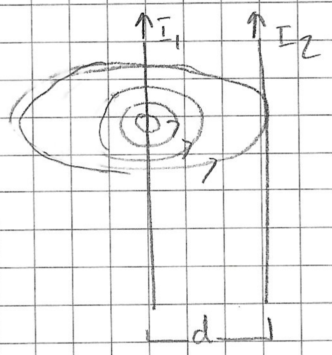
Which one has the stronger field at the center?

first one.

Force between two parallel wires

- A current carrying wire experiences force in a magnetic field
- A current carrying wire produces a magnetic field

⟶ Two current carrying wires will exert force to each other



The magnetic field on second wire due to first wire is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

The force exerted on second wire due to first wire's magnetic field

$$F_2 = I_2 l_2 B_1 = \boxed{\frac{\mu_0}{2\pi}\frac{I_1}{d} I_2 l_2}$$ (parallel wires)

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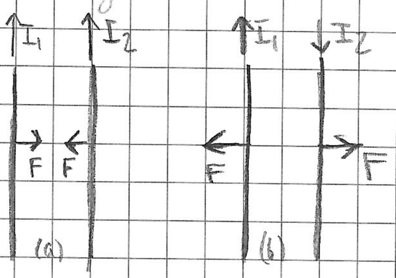
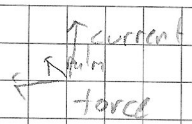
right hand rule





Parallel currents in the same direction attract each other
Parallel currents in opposite direction repel each other

If two wires in an appliance cord are 3.0 mm apart and carry 8.0 A, calculate the force they exert on each other

$$F = \frac{(4\pi \times 10^{-7}\ Tm/A)\ (8.0A)^2\ (2.0m)}{(2\pi)\ (3.0 \times 10^{-3} m)} = 8.5 \times 10^{-3} N$$

Definitions of the Ampere and the Coulomb

Ampere is defined through magnetic field it produces

if $I_1 = I_2 = 1A$ and the wires are 1 m apart

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{(4\pi \times 10^{-7}\ T \cdot m/A)}{(2\pi)}\frac{(1A)(1A)}{1m}$$

$$= 2 \times 10^{-7}\ N/m$$

One Ampere → the current flowing in each of two long wires which results in a force of exactly 2×10^{-7} N per meter

→ μ_0 is fixed from this

......./......./.......

→ Coulomb is defined as one Ampere-Second

$$1\,C = 1\,A.s$$

→ The value of ε_0 is obtained from experiment

This is about minimizing the errors, i.e.
it is hard to determine coulomb directly from experiment, as charges easily leak

this is called operational definition
- definitions of quantities that can exactly be measured given a set of operations

the amount of current in a wire can be adjusted precisely and continuously → less error

→ Calibration and Standards are a power!

Electric and Magnetic fields are defined operationally

$$\vec{E} = \vec{F}/q$$ (measurable force on a charge)

$$B = \frac{F_{max}}{I\ell}$$ (measurable force on a current carrying wire)

Ampère's Law

Andre-Marie Ampere

one of the founders of classical electromagnetism

inventor of solenoid and numerous other applications

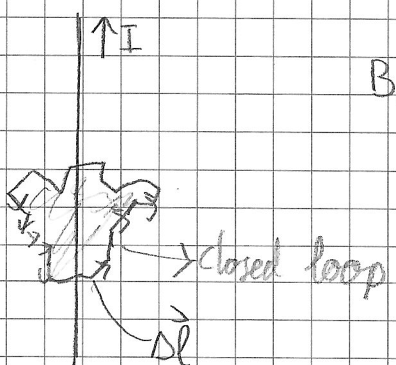
inventor of telegraph

one of the 72 names inscribed in Eiffel tower

- father was killed by the jacobins 1793

  jacobins: nazis of france, declared war on Austria and Pruss

- Success in Napelon's technocracratic first consul
- 1802 left his ailing wife to become professor in dyon → game theory
- 1803 moved to paris to École polytechnique



$B_{\parallel}$ → magnetic field's parallel component to $\Delta \vec{l}$

$$\sum B_{\parallel} \Delta l = \mu_0 I_{encl}$$

$$\boxed{\oint \underbrace{\vec{B} \cdot}_{B_{\parallel}} d\vec{l} = \mu_0 I_{encl}}$$ Ampère's law

I_{encl}: the current passing through the area enclosed

$\Delta \vec{l} \to d\vec{l}$

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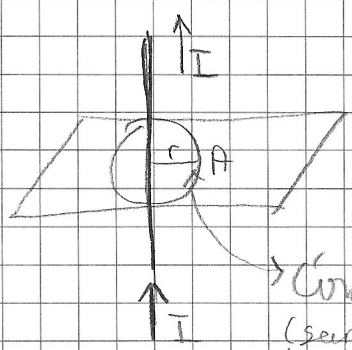
Single long straight wire carrying current I



$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = \oint B\, dl = B\oint dl = B 2\pi r$$

$$B = \frac{\mu_0 I}{2\pi r} \rightarrow \text{same formula}$$

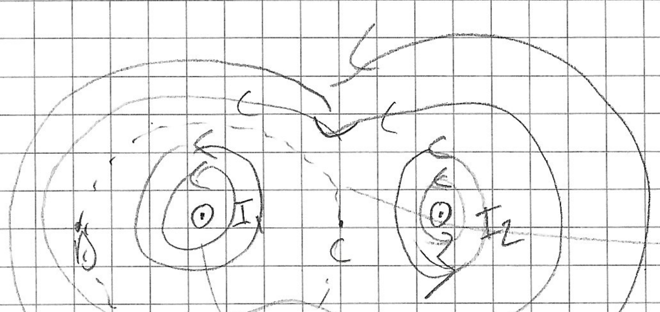
→ Circular path
(same distance → $|B_\perp|$ same along path)
(symmetry → $B_\parallel = |\vec{B}|$)

Like the Gauss's law Ampere's law has very limited practical use for calculating $\vec{B}$

→ It is a basic law that relates current and $\vec{B}$ directly

→ Valid when no magnetic material is present and I is steady

$B = \frac{\mu_0}{2\pi} \frac{I}{r}$ → this is so, because we want to write $\mu_0 I$ in Ampere's law

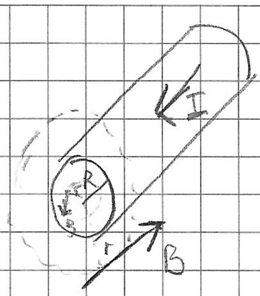


Notice that $\vec{B}$ around I_1 is both due to I_1 and I_2

however $\oint \vec{B} \cdot d\vec{l}$ here is still $\mu_0 I_1$ no matter I_2 is present or not

→ $\vec{B}$ changes $\oint \vec{B} \cdot d\vec{l}$ stays the same!!

→ C: $B = 0$ D: $B > B_0$



A long straight cylindrical wire conductor of radius R carries I with uniform current dens.

Choose a circular path due to symmetry

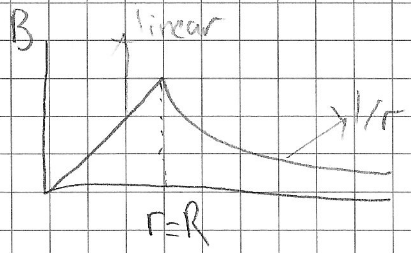
$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl}$$

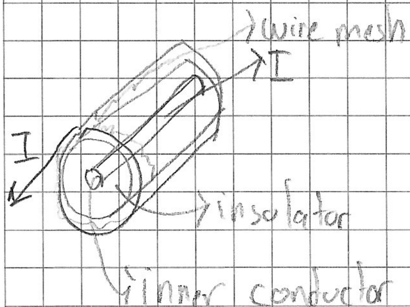
outside $I_{encl} = I \rightarrow B = \dfrac{\mu_0 I}{2\pi r}$

Inside $I_{encl} = \dfrac{I\pi r^2}{\pi R^2} \rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

$$B(2\pi r) = \mu_0 I \left(\frac{\pi r^2}{\pi R^2}\right)$$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

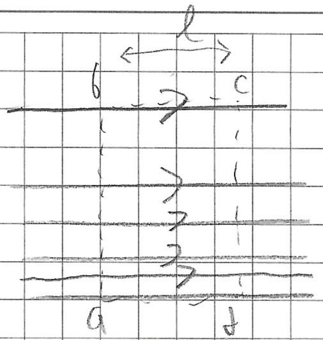


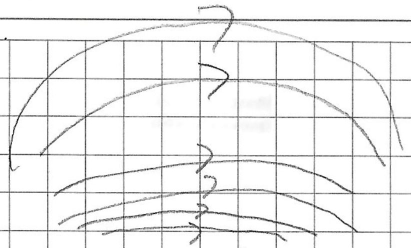


what is B in between the wire and outside the wire?

→ In between → only inner current matters

→ Outside currents cancel → magnetic shield





can we have an inhomogeneous $\vec{B}$ field like this?

↑ this is possible

no!

$$\oint \vec{B} \cdot d\vec{l} = 0 \quad \text{(no current)}$$

$$= B_{bc} l - B_{da} l \neq 0 \quad \text{: for (1)}$$

can be 0 for (2)

Ampere's law

- Always valid, not always handy
- When there is symmetry, identify a path that exploits it.
  - Constant B
  - make sure it passes through probe point
- Determine direction of $\vec{B}$, try to keep it $\parallel$ or $\perp$ to path at all times by choosing an appropriate path
- Determine I_{encl}
  - Be careful with signs
  - Use right hand rule
  - Use current density

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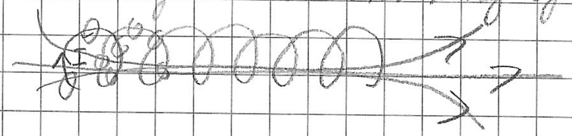
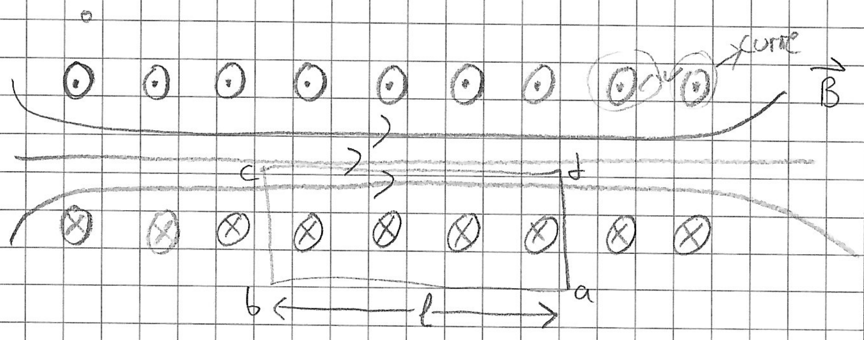
Chapter 28 PS V1 ....../....../.......

Magnetic field of a solenoid and a toroid

Solenoid: A long coil of wire consisting of many loops



- Between the wires, fields tend to cancel.
- Toward the center, fields add up
- For a long solenoid with closely packed coils the field is nearly uniform and parallel to its axis
- Since same number of field lines are compressed in a small volume, magnetic field inside is much larger than outside



$$\oint \vec{B}\cdot dl = \underbrace{\int_a^b \vec{B}\cdot d\vec{l}}_{\text{near zero}} + \underbrace{\int_b^c \vec{B}\cdot d\vec{l}}_{\text{near zero}} + \int_c^d \vec{B}\cdot d\vec{l} + \underbrace{\int_d^a \vec{B}\cdot dl}_{\text{near zero}}$$

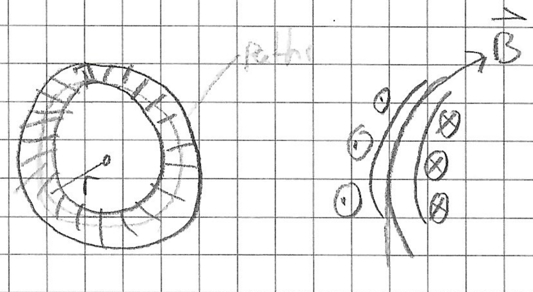
$$\oint \vec{B}\cdot d\vec{l} = \int_c^d \vec{B}\cdot d\vec{l} = Bl$$

If current is I and there are N loops

$$Bl = \mu_0 N I \rightarrow \mu_0 n I = B \quad \text{where } n = \frac{N}{l}$$

B depends only on the number of loops per length and I, it is position independent, thus, uniform

Toroid



inside the toroid: $\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$

$\Rightarrow B\, 2\pi r = \mu_0 N I \longrightarrow B = \frac{\mu_0 N I}{2\pi r}$

notice the r dependence! → B is more intense towards inner edge! → thin toroid → similar to solenoid

$B = \mu_0 n I$ where $n = \frac{N}{2\pi r}$ (thin!)

outside the toroid → net current is zero!
(same # of in as out)

$\oint \vec{B} \cdot \vec{dl} = 0 \rightarrow B = 0$

All the field is inside the loop

Biot - Savart Law

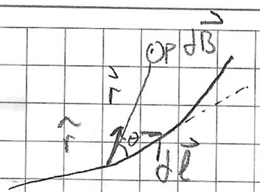
Like Gauss's law, Ampere's law is fundamental but its use for calculating $\vec{B}$ is limited to symmetric cases.

Jean Baptise Biot and Felix Savart 1820

↑ metorite
biotite
early baloon flight
climbed 7 010 meters
to proove magnetic field
declines (very dangerous)

↑ acoustics, vibrations
violins (savart music interval measure)
range of human hearing

$$\boxed{d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}}$$



Biot-Savart Law

$$dB = \frac{\mu_0 I \, dl \sin\theta}{4\pi r^2}$$

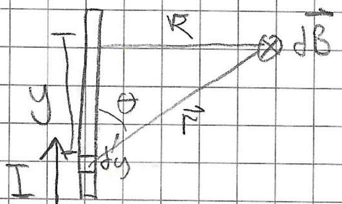
$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^2} \quad \text{Vector sum !!!}$$

similar to Coulomb's law

$\vec{B}$ in Biot-Savart law is only due to I along $d\vec{l}$ nothing else

To find total $\vec{B}$, one must count all currents

$\vec{B}$ due to straight wire



$$B = \frac{\mu_0 I}{4\pi} \int_{y=-\infty}^{y=+\infty} \frac{dy \sin\theta}{r^2}$$

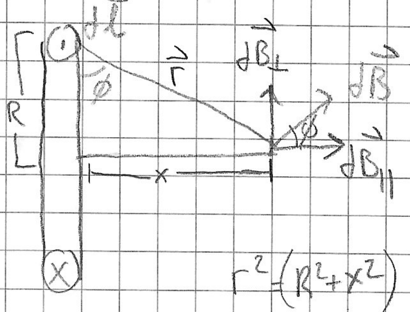
$$dy = dl \qquad r^2 = R^2 + y^2 \qquad y = -R/\tan\theta$$

$$dy = +R\csc^2\theta \, d\theta = \frac{R\,d\theta}{\sin^2\theta} = \frac{R\,d\theta}{(R/r)^2} = \frac{r^2 d\theta}{R}$$

$$y = -\infty \rightarrow \theta = 0 \quad y = +\infty \quad \theta = 180^\circ$$

$$B = \frac{\mu_0 I}{4\pi} \frac{1}{R} \int_{\theta=0}^{\pi} \sin\theta \, d\theta = -\frac{\mu_0 I}{4\pi R} \cos\theta \Big|_0^{\pi} = \frac{\mu_0 I}{2\pi R}$$

$\vec{B}$ due to a current loop



Symmetry $\rightarrow \vec{B}_\perp$ will cancel out

$$B = B_\parallel = \int dB\cos\phi = \int dB \frac{R}{r} = \int dB \frac{R}{(R^2+x^2)^{1/2}}$$

$$r^2 = (R^2 + x^2)$$

$$= \frac{\mu_0 I}{4\pi} \frac{R}{(R^2+x^2)^{3/2}} \underbrace{\int dl}_{2\pi R} = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$

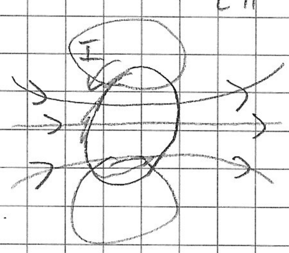
at $x = 0$ (center) $B = \frac{\mu_0 I}{2R}$

......./......./.......

recall the magnetic dipole $\mu = N I A$ ← area, ← # of loops

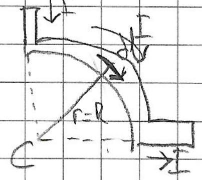
from previous solution: $B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} = \frac{\mu_0}{2\pi} \frac{\mu}{(R^2+x^2)^{3/2}}$

$\cup$ $B \simeq \frac{\mu_0}{2\pi} \frac{\mu}{x^3}$ (on axis magnetic dipole $x \gg R$)



magnetic field due to a current loop

$\vec{B}$ due to wire segment



straight sections $\to d\vec{l} \times \hat{r} = 0$

$dB = \frac{\mu_0 I \, dl}{4\pi R^2}$ $(\sin\theta = 1)$

$\hat{r} \perp dl$

$B = \int dB = \frac{\mu_0 I}{4\pi R^2} \int dl = \frac{\mu_0 I}{4\pi R^2} \left(\frac{1}{4} 2\pi R \right) = \frac{\mu_0 I}{8R}$