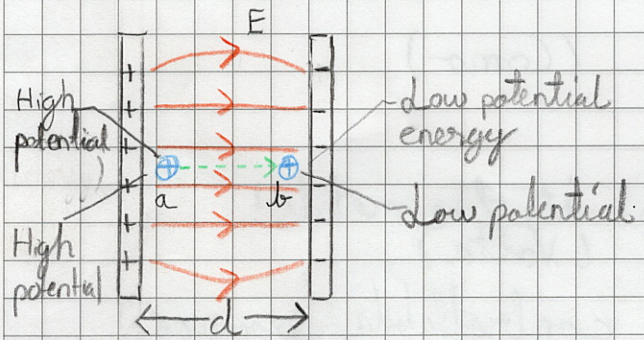
Electric Potential Energy and Potential difference

Electrostatic force between two charges is conservative, since it goes $\propto 1/r^2$ work done in moving a particle between two points is independent of the path taken

→ We can define a potential energy term and use energy point of view!
conservation of mechanical energy

The change in potential energy between two points a and b equals to the negative of the work done by the conservative force



$$\Delta U = -W$$

For example, a small + charge like described on the figure will move towards the − plane. The work done by the electric field by moving a distance d from a to b is

$$W = Fd = qEd$$

The change in potential energy is − of this work (since we needed effort to put + charge in a)

$$U_b - U_a = -W = -qEd$$

i.e potential energy decreased from a to b

$$\Delta KE\uparrow + \Delta PE\downarrow = 0$$

Potential (Electric potential and potential difference)

(Electric) Potential : electric potential energy per unit charge

$$V_a = \frac{U_a}{q}$$

→ Since only the differences in potential energy has physical meaning, only differences in the potential has a physical meaning

$$V_{ba} = \Delta V = V_b - V_a = \frac{U_b - U_a}{q} = \frac{-W_{ba}}{q}$$

The unit of electric potential : $1 \frac{\text{Joule}}{\text{Coulomb}}$ = Volts

→ Alessandro Volta 1745-1827

Italian physicist / chemist (Como)

- battery - methane

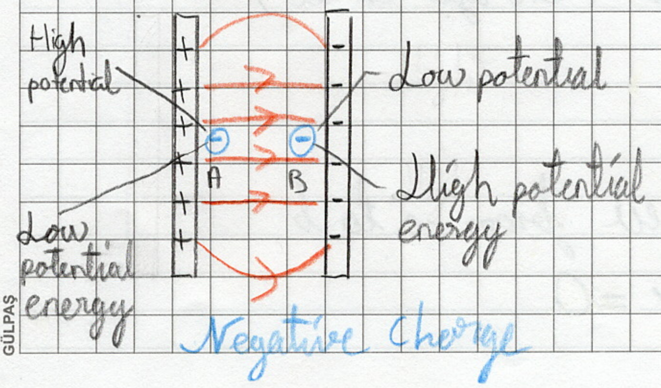
animal electricity (Galvani) no! its the metals! (Volta)

he created a stable and controllable source of electric current ; Voltaic pile

→ electrochemistry (seperation of H_2O)

Potential (Voltage) at a point depends on the reference

Often the ground is taken as zero, In some other cases ∞ is zero.



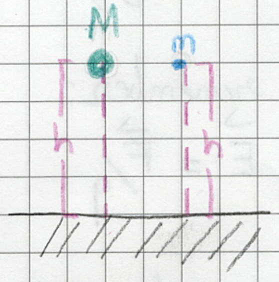
Potential does not depend on the charge of the probe (potential energy does!)

here ΔU and ΔV has the opposite sign!

Chapter 23 P2 V2 ....../....../......

An analogy for understanding the difference between potential and potential energy

$$\text{"gravitational potential"} : \frac{\text{gravitational potential energy}}{\text{mass}}$$



- Both mass M and mass m has the same "gravitational potential" $= gh$
- mass $M >$ mass $m \rightarrow$ gravitational potential energy of mass M is greater

Batteries and generators provide potential difference. The energy depends on the current you can draw from them

Some Typical Potential differences

Thundercloud to ground	$\sim 10^8\,V$
High voltage power line	$\sim 10^5 - 10^6\,V$
Auto ignition	$\sim 10^4\,V$
220V AC	$\sim 10^2\,V$
Auto battery	$\sim 12\,V$
AA battery	$\sim 1.5\,V$
Nerve membrane	$\sim 10^{-1}\,V$
Potential changes in skin (EEG) (EKG)	$\sim 10^{-4}\,V$

can shock humans

Relation between Electric Potential and Electric field

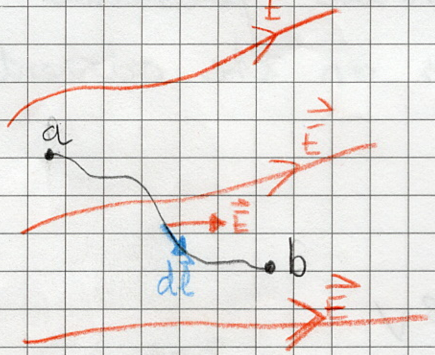
The effects of any charge distribution can be described either in terms of electric field (force view) or electric potential (energy view)

$$\frac{U_b - U_a}{q} = \frac{-\int_a^b \vec{F} \cdot d\vec{l}}{q}$$

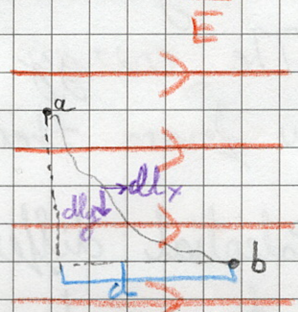
remember: $\vec{E} = \vec{F}/q$

$$\boxed{V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}}$$

general case



general case



special case (uniform $\vec{E}$)

when $\vec{E}$ is uniform

$$V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l}$$

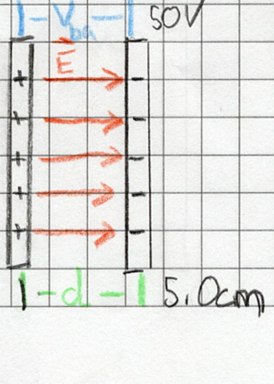
$$= -E \int_a^b dl$$

$$\boxed{V_{ba} = -Ed}$$

Special case (uniform $\vec{E}$)

notice that $\frac{V}{m} = \frac{J}{mC} = \frac{N}{C}$ unit of $\vec{E}$!

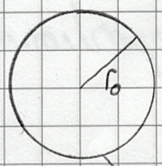
example: What is the $|\vec{E}|$ between the plates of a parallel plate capacitor held at 50V which has plates $d = 5.0$ cm apart?



parallel plate field: uniform $\vec{E}$

$$E = \frac{V_{ba}}{d} = \frac{50\,V}{0.050\,m} = \boxed{1000\ V/m}$$

Charged conducting sphere



conducting sphere (total charge = Q)

Determine the potential

a) $r > r_0$ b) $r = r_0$ c) $r < r_0$

→ We know $\vec{E}$ due to a spherical charge distribution is also spherically symmetric

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q_{encl}}{r^2}\hat{r}$$

a) $r > r_0$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q_{encl}}{r^2} \qquad V_b - V_a = -\int \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2}$$

↳ use spherical coordinates r, θ, ϕ

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

in order to set the potential at r, we need a reference point. In these kinds of problems we set V=0 at infinity. set $r_b = \infty$ $r_a = r$

$$\cancel{V_b}^{\,0} - V_a = \frac{Q}{4\pi\epsilon_0}\left(\underbrace{\cancel{\frac{1}{r_b}}}_{\infty}^{\,0} - \underbrace{\frac{1}{r_a}}_{r}\right) \longrightarrow V = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

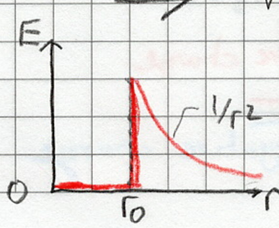
b) at $r = r_0$

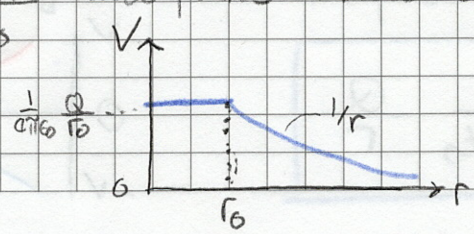
$$V = \frac{Q}{4\pi\epsilon_0}\frac{1}{r_0}$$

c) Within the conductor $E = 0$

$$V_b - V_a = 0$$

$$\longrightarrow V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0} \quad \text{(all points inside the conductor)}$$





example : Breakdown voltage

Air becomes a conductor above 3×10^6 V/m

a) Show that the breakdown voltage is proportional to the radius of a charged sphere

b) What is the max potential you can achieve for a sphere of $R = 1.0$ cm in air?

things we know about spheres

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0} \qquad E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2}$$

a) $V = r_0 E$ at the surface (see previous ex.)

b) $r_0 = 5\times10^{-3}$ m $\rightarrow$ $V = (5\times10^{-3}\text{ m})(3\times10^6\text{ V/m}) \cong 15000$ V

Corona discharge : High E near sharp points ionize air. Light comes out when $\bar{e}$ move down to their ground state

Lightning rods : They ionize surrounding air to provide a conduction path to lightning

Electric potential due to point charges (Coulomb potential)

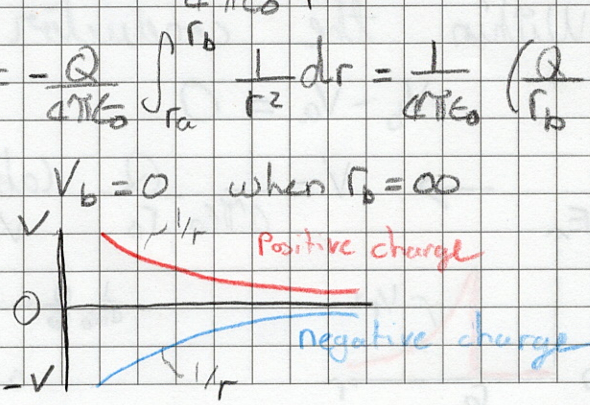
$$V_b - V_a = -\int \vec{E}\cdot d\vec{l}$$

for a point charge : $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ (spherically sym)

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E}\, dl = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}dr = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

reference point : set $V_b = 0$ when $r_b = \infty$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}}$$



work required to bring two positive charges close

$q = 3.0\,\mu C$ take q, move it from $r=\infty$ to i.e.

$Q = 20.0\,\mu C$ $r = 0.5m$

$$W = \Delta U = \underbrace{q}_{\text{probe}}\underbrace{(V_b - V_a)}_{\text{potential source}} = q\left(\frac{kQ}{r_b} - \frac{kQ}{r_a}\right)$$

$r_b = 0.5m$ $r_a = \infty$

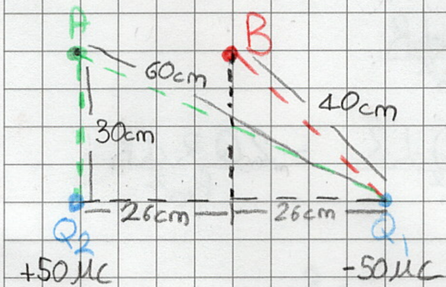
$$W = (3.0\times10^{-6}C)\ \frac{(8.99\times10^9\,Nm^2/C^2)(2.00\times10^{-5}C)}{0.5m} = \boxed{1.08J}$$

• Why not $V_{ba} = -Ed$? field is not uniform

• Why not $W = Fd$? $\vec{F}$ is not constant (due to field)

Energy point of view is much more easier since we deal with scalars instead of vectors

Example: Potential above two charges



$$V_A = V_{A2} + V_{A1} = \frac{kQ_2}{r_{2A}} + \frac{kQ_1}{r_1 a}$$

$$= (9\times10^9\,Nm^2/C^2)\left(\frac{5\times10^{-5}C}{0.3m} + \frac{(-5\times10^{-5}C)}{0.6m}\right)$$

$$= 7.5\times10^5\ V$$

$$V_B = V_{B1} + V_{B2} = \left(9\times10^9\frac{Nm^2}{C^2}\right)\left(\frac{50\mu C - 50\mu C}{0.4m}\right)$$

$$= 0\,V$$

Potential due to an arbitrary charge distribution

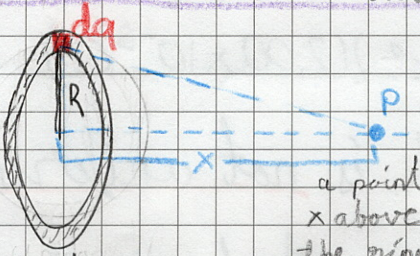
When $\vec{E}$ is known $\rightarrow$ $V_{ba} = -\int_a^b \vec{E}\cdot d\vec{l}$

more often than not $\vec{E}$ is unknown and very difficult to obtain. If you don't need the direction of the force anywhere, use Coulomb potential!

$$V_a = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{Q_i}{r_{ia}}$$

$$\rightarrow \boxed{V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}}$$

Example: Potential due to a ring charge



a point x above the ring

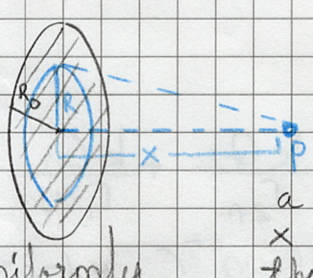
uniformly charged thin ring

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{1}{(x^2+R^2)^{1/2}} \int dq$$

$$\boxed{= \frac{1}{4\pi\epsilon_0} \frac{Q}{(x^2+R^2)^{1/2}}}$$

notice: $\lim x >> R \rightarrow$ point charge

Example: Potential due to a disc charge



a point x above the disk

uniformly charged thin disk

Total disk area $= \pi R_0^2$

each thin ring area $= 2\pi R dR$

$$\frac{dq}{Q} = \frac{2\pi R\, dR}{\pi R_0^2}$$

$$dq = \frac{Q(2\pi R)dR}{\pi R_0^2} = \frac{2QR\,dR}{R_0^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{(x^2+R^2)^{1/2}} = \frac{2Q}{4\pi\epsilon_0 R_0^2} \int_0^{R_0} \frac{R\,dR}{(x^2+R^2)^{1/2}}$$

$$= \frac{Q}{2\pi\epsilon_0 R_0^2} (x^2+R^2)^{1/2} \Big|_{R=0}^{R=R_0}$$

$$\boxed{= \frac{Q}{2\pi\epsilon_0 R_0^2} \left[(x^2+R_0^2)^{1/2} - x \right]}$$

check: $x >> R_0 \rightarrow$ point charge ✓

Equipotential surfaces

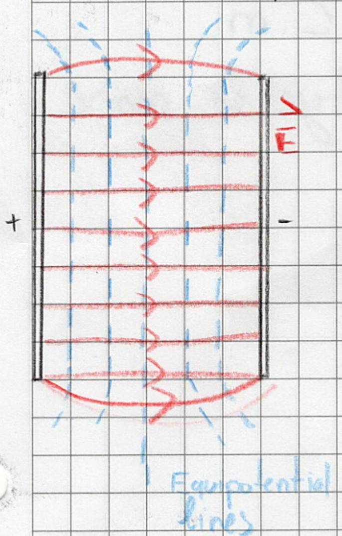
Electric potential can be represented graphically for analysis.

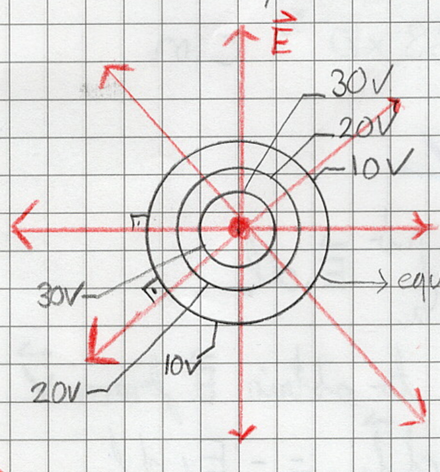
→ collect points in space where the potential is equal, together

2D → equipotential lines

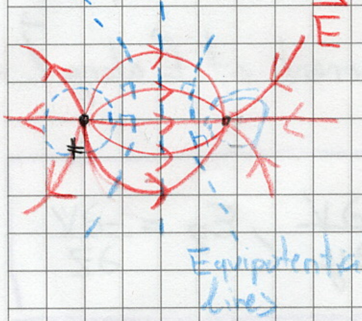
3D → equipotential surfaces

Consequence: $\vec{E}$-field lines are always perpendicular to equipotential lines / surfaces.
(since parallel component has $\Delta V = 0$)





• $\vec{E}$ points towards lower values of V



Equipotential lines / surfaces are always continious and never terminate (so they continue beyond what is drawn on the left)

A conductor must be entirely at the same potential in the static case (otherwise $\bar{e}$ will move to cancel it) hence is equipotential

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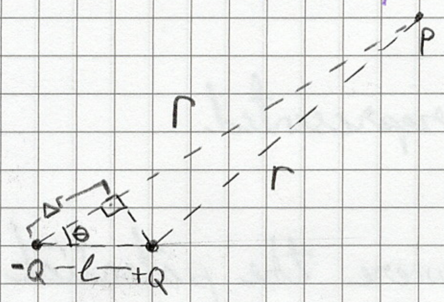
Electric dipole potential



Two equal point charges Q of opposite sign seperated by distance l

The electric potential at point P is
(V=0 at r=∞)

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} + \frac{1}{4\pi\epsilon_0}\frac{(-Q)}{(r+\Delta r)} = \frac{1}{4\pi\epsilon_0}Q\left(\frac{1}{r} - \frac{1}{r+\Delta r}\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{\Delta r}{r(r+\Delta r)}$$

when $r \gg l$ $\quad \Delta r \sim l\cos\theta$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Ql\cos\theta}{r.r} = \boxed{\frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}} \text{ dipole } r \gg l$$

$p = Ql$ is the dipole moment has units C.m

when dealing with molecules etc, debye is more used $\quad$ 1 debye $= 3.33 \times 10^{-30}$ C.m

$\vec{E}$ determined from V

$$V_b - V_a = -\int_a^b \vec{E}.d\vec{l}$$

invert to obtain $\vec{E}$ from $\vec{V}$

$$dV = -\vec{E}.d\vec{l} = -E_l\, dl$$

$\hookrightarrow$ component of $\vec{E}$ along $\vec{l}$

$$E_l = -\frac{dV}{dl}$$

partial derivative

$$\boxed{E = -\frac{dV}{dl}} \text{ when } \vec{E} \parallel \vec{l} \qquad \boxed{E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}}$$

i.e: $V(x,y,z) = (2\,V/m^2)\,x^2 + (8\,V/m^3)\,y^2 z + (2\,V/m^2)\,z^2$

$$E_x = -\frac{\partial V}{\partial x} = -(2(2V/m^2)x + 0 + 0) = -(4\,V/m^2)\,x$$

$$E_y = -\frac{\partial V}{\partial y} = -(0 + 2(8V/m^3)\,yz + 0) = -(16\,V/m^3)\,yz$$

$$E_z = -\frac{\partial V}{\partial z} = -(0 + (8\,V/m^3)\,y^2 + 2(2\,V/m^2)\,z) = -8\,V/m^3\,y^2 - 4\,V/m^2\,z$$

Example: $\vec{E}$ for a ring and a disk

Use the electric potential to calculate $\vec{E}$ at a point on the axis of a a) ring b) disk

– both V has been calculated previously

a) $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{(x^2+R^2)^{1/2}}$

$\rightarrow E_x = \frac{-\partial V}{\partial x} = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2+R^2)^{3/2}}$

b) $V = \frac{Q}{2\pi\epsilon_0 R_0^2} \left[(x^2+R_0^2)^{1/2} - x \right]$

$E_x = -\frac{\partial V}{\partial x} = \frac{Q}{2\pi\epsilon_0 R_0^2} \left[1 - \frac{x}{(x^2+R_0^2)^{1/2}} \right]$

when very close to the disk = plane

$x \ll R_0$ $\qquad E_x = \frac{Q}{2\pi\epsilon_0 R_0^2} = \frac{\sigma}{2\epsilon_0} \qquad \sigma = \frac{Q}{\pi R_0^2}$

Electrostatic Potential Energy ; the Electron Volt

Suppose a point charge q is moved between two points in space, a and b, where the electric potential due to other charges V_a and V_b, respectively

$$\Delta U = U_b - U_a = q(V_b - V_a)$$

What is the electrostatic potential energy of a charge ensemble?

Energy required to bring two charges from infinity together

$$U = Q_2 V = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{12}}$$

V=0 r=∞

Energy required to bring three charges together

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right)$$

four charges → six terms ....

Joule is very large for microscopic world we use eV. eV units are not SI (must be converted)

$$1\,eV = 1.6\times10^{-19}\,J$$

ex: Hydrogen atom ionisation energy

average radius $\underbrace{0.529\times10^{-10}\,m}_{\text{Bohr radius}}$

$$U = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{(e)(-e)}{r} = -\frac{(8.99\times10^{9}\,N\,m^2/C^2)(1.6\times10^{-19}\,C)^2}{0.529\times10^{-10}\,m}$$

$$= -27.2\,(1.6\times10^{-19})\,J = -27.2\,eV$$

Virial theorem → $U = -\frac{1}{2}K$

$$E = \underset{13.6eV}{K} + \underset{-27.2eV}{U} = -13.6\,eV \leftarrow \text{Rydberg energy}$$

Earnshaw's Theorem (Samuel Earnshaw 1842)

→ A collection of point charges can not be maintained in a stable stationary eq. solely by the electrostatic interaction of charges

→ electrons couldn't be moving in an atom due to electromagnetic losses

→ How can the atom be?

KE term is "strange" → Quantum theory