1.75 fm = 1.75×10^{-15} m. 5.29 $\times 10^{-11}$ m 10.000 times

- pushes and pulls, friction,
- forces that hold matter together as liquids, solids...

are all electric forces acting on molecular level

21.1

Amber effect → rub a cloth → small pieces of leaves and dust get attracted

→ Static electricity

- combing your hair
- nylon carpet
- syntethic clothes

In each case, the object becomes charged

- It posesesses a net electric charge

- How many types of electric charge exist?

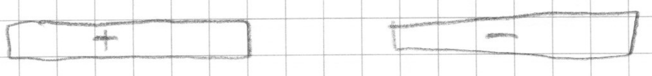
Two

unlike charges attract; like charges repel

We need a convention, a naming scheme

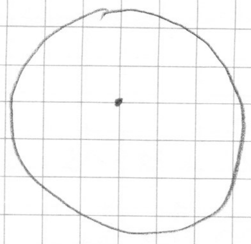
Benjamin Franklin convention

glass rod amber



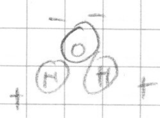
- Charge has to be treated algebraicly
  - net charge produced is zero
  - no net electric charge can be created or destroyed

21.2 Past century: Atom → electric charge connection



when an atom has net positive or negative charge, it is called an ion

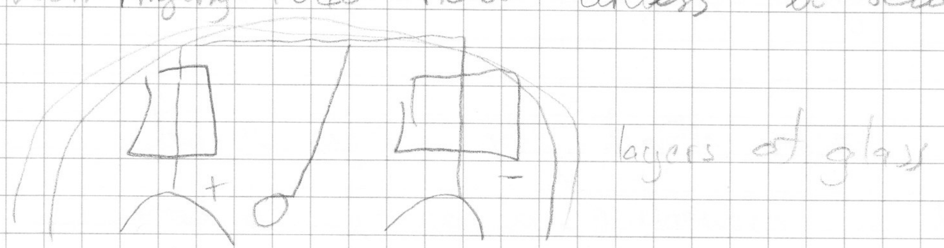
when the charge is not distrubuted evenly, the molecule is polar



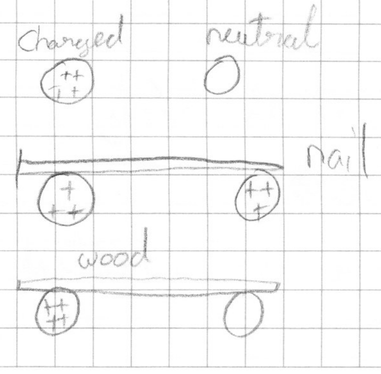
because water is polar, charged objects leak off their charge to the air

Oxford Electric bell

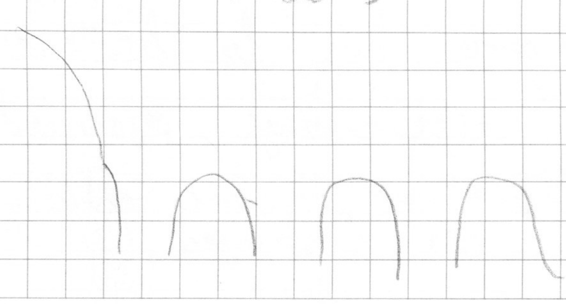
has been ringing 1840—now unless it becomes too humid



21.3 Insulators Conductors and Semi-conductors

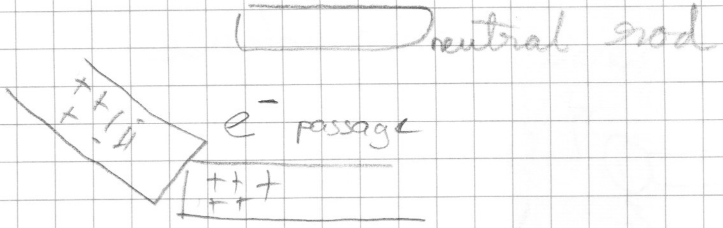


Semiconductors

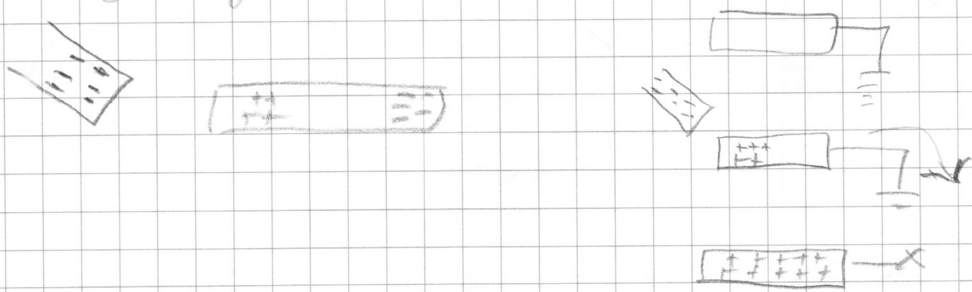


21.4

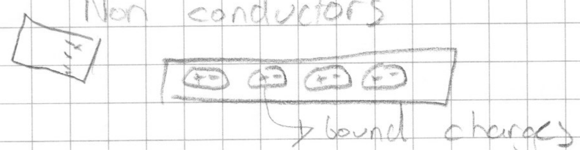
- Charge by contact = charge by conduction



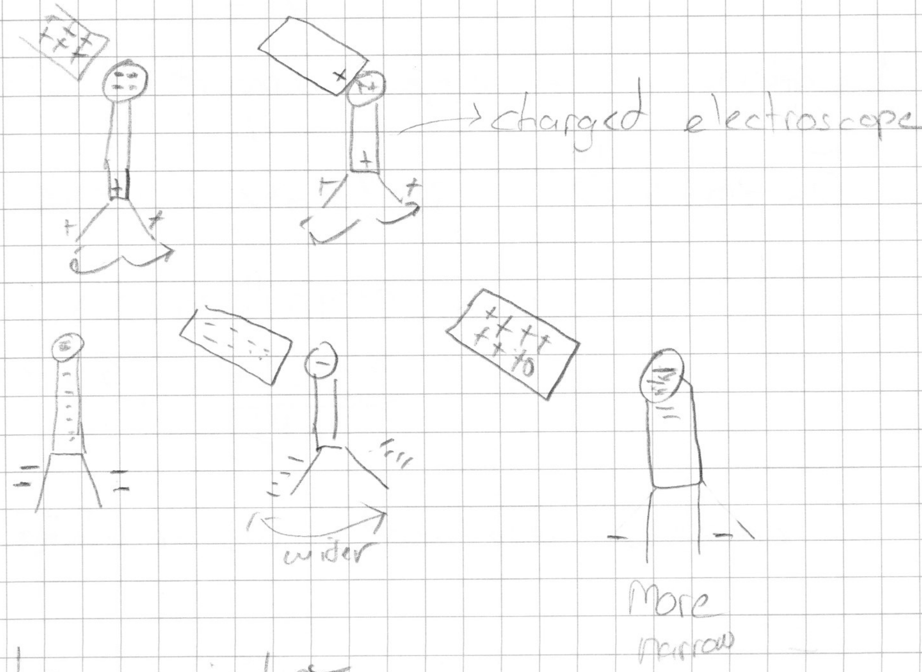
- Charge by induction



Non conductors



electroscope!

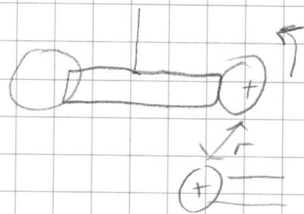


electrometer : modern equivalent

21.5 Coulomb's law

Charles coulomb 1736–1806

1780s using a torsion balance



- exact charge unknown
- the ratio of charges are known

(+) O OO
1/2 1/2

$$F_{12} = k \frac{Q_1 Q_2}{r^2}$$ $Q_1 \longleftarrow r \longrightarrow Q_2$ (coulomb's law)

r^2 → experimental parameter accurate 2 ± 10^{-16}

SI unit of charge is coulomb (C)

$k = 8.99 \times 10^9 \ N.m^2/C^2 \approx 9 \times 10^9 \ N.m^2/C^2$

1 Coulomb is a huge amount of charge!

1C ←1m→ 1C

$9 \times 10^9 \ N m^2/\cancel{C^2} \ \frac{1\cancel{C}\ 1\cancel{C}}{1 m^2} = 9 \times 10^9 N$

~ million ton weight

$\frac{9 \times 10^9 N}{10 N/kg} = 9 \times 10^8 kg \approx$ million ton

charges produced by rubbing $\approx 1\mu C = 10^{-6} C$

- Positive charge : deficit of electrons
- negative charge : excess of electrons

charge of electron : $-e$

$$e = 1.602 \times 10^{-19}\ C$$

net charge on any object : an integral multiple of e

electric charge is quantized
(exist only in discrete amounts)

e is small $\longrightarrow$ macroscopic scale charging seems continious

$$1 \mu C \approx 10^{13} \text{ electrons}$$

Permittivity of free space :

$$k = \frac{1}{4\pi\epsilon_0}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} \quad \text{(coulomb's law)}$$

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ C^2/N.m^2$$

ϵ_0 is more handy than k due to maxwell's equations

note that in the book's notation parenthesis are implicit so above units read $C^2/(N.m^2)$

Coulomb's law is precise for "point charges"

Where can we assume point charge?

- when the distance between objects are much larger than the size of the object
- when symmetry allows it (i.e a homogenously charged sphere can be considered as a point charge at origin)

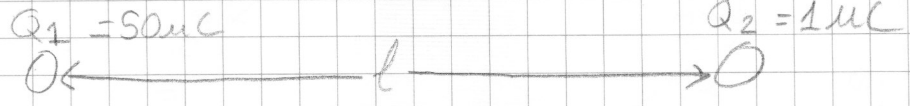
otherwise we need to incorporate how the charge is distrubuted to our solution, However the principle remains the same

Coulomb's law describes the force between two charges when they are at rest

- There are additional forces that come into play when charges move

In this chapter we focus on electrostatics and electrostatic force

Tip: you can use coulomb's law for magnitude then use vector analysis seperately to determine the direction which charge exerts greater force

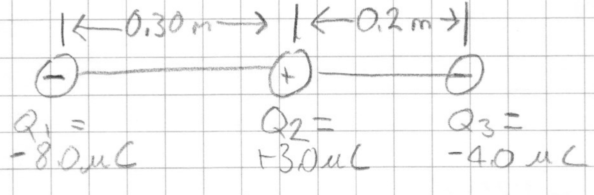


$$F_{12} = k\frac{Q_1 Q_2}{l^2}$$

$$F_{21} = k\frac{Q_2 Q_1}{l^2}$$

$F_{12} = F_{21}$ (Newton's third law)

<u>Principle of superposition</u>;



what is the net electrostatic force on Q_3?

$\vec{F} = |F|\hat{x}$

$F = -F_{32} + F_{31}$

$$F_{31} = k\frac{Q_3 Q_1}{r_{31}^2} = \frac{(9.10^9\ Nm^2/C^2)(4.0\times10^{-6}C)(8.0\times10^{-6}C)}{(0.50m)^2}$$

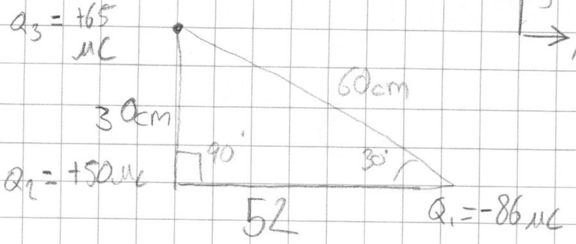
$= 1.2N$

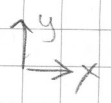
$$F_{32} = k\frac{Q_3 Q_2}{r_{32}^2} = \frac{(9.10^9\ Nm^2/C^2)(4\times10^{-6}C)(3\,10^{-6}C)}{(0.2\ m)^2}$$

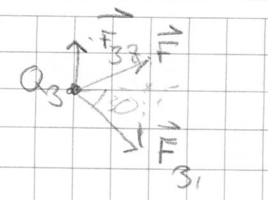
$= 2.7N$

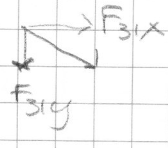
$F = -2.7 + 1.2N = -1.5N$

The principle of superposition; the electrostatic force Q_1 exerted on Q_3 is not affected by the presence of Q_2









$$F_{31} = k\frac{Q_3 Q_1}{r_{31}^2} = \frac{(9\times10^9\,Nm^2/C^2)(6.5\times10^{-5}C)(8.6\times10^{-5}C)}{(0.6m)^2}$$

$$= 140N$$

$$F_{32} = k\frac{Q_3 Q_2}{r_{32}^2} = \frac{(9\times10^9\,Nm^2/C^2)(6.5\times10^{-5}C)(5.0\times10^{-5}C)}{(0.30\,m)^2}$$

$$= 330\,N$$

$$\vec{F}_{31} = F_{31x}\,\hat{x} + F_{31y}\,\hat{y}$$

$$F_{31x} = F_{31}\cos 30 = 120\,N$$

$$F_{31y} = -F_{31}\sin 30 = -70\,N$$

$$\vec{F} = F_x\,\hat{x} + F_y\,\hat{y}$$

$$F_x = F_{31x} = 120N$$

$$F_y = F_{32} + F_{31y} = 260N$$

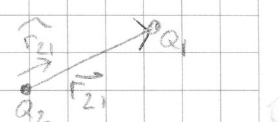
angle with $\hat{x}$ (θ) = $\tan\theta = \frac{F_y}{F_x} = \frac{260N}{120N} = 2.2$

$$\theta = 65^\circ$$

Note

to save time on drawing you can also use vector notation

$$\vec{F}_{12} = k\frac{Q_1 Q_2}{r_{21}^2}\,\hat{r}_{21} \longrightarrow \text{from } Q_2 \text{ to } Q_1$$



→ Vector force on charge Q_1 due to Q_2

if Q_2 and Q_1 has same sign $\vec{F}_{12}$ is along $\vec{r}_{21}$: repulsive

21.6 The Electric field

The field concept is immensly useful for taking advantage of superposition principle

It allows mixing and matching well known solutions for unique problems

- A "test charge" is an infinitesmall positive charge that does not exert force to the charge distribution it is probing
- Electric field is the force exerted on the test charge

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{where } q \to 0$$

$$E = \frac{F}{q} = \frac{kqQ/r^2}{q}$$

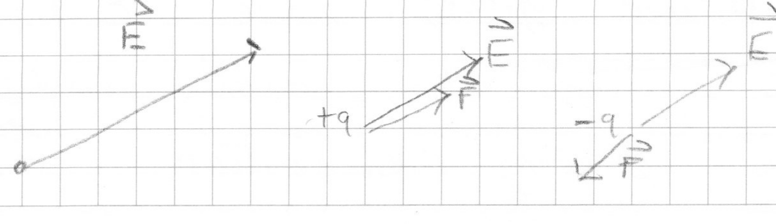
$$= k\frac{Q}{r^2}$$ → a property of the probed charge only! very useful

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

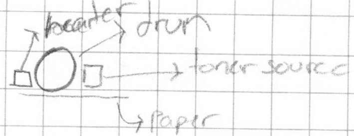
- A use of the Electric field:

The force exerted on charge q placed on an electric field is

$\vec{F} = q\vec{E}$ (as long as q does not change E)



Laser printer



In order to temporarily bind toner to the drum they must overcome twice their weight

each toner particle $= 9 \times 10^{-16}$ kg
charge on toner $= -20e$

$$qE = 2mg$$

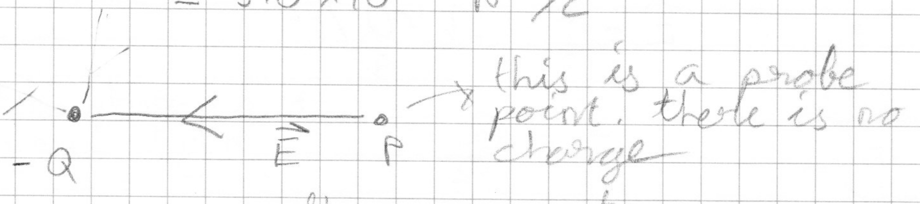
$$E = \frac{2mg}{q} = \frac{2(9 \times 10^{-16}\,\text{kg})(9.8\,\text{m/s}^2)}{20(1.6 \times 10^{-19}\,\text{C})} = 5.5 \times 10^{3}\,\text{N/C}$$

Electric field of a single point charge

$$Q = -3.0 \times 10^{-6}\,\text{C}$$

$$E = k\frac{Q}{r^2} = \frac{(9.0 \times 10^{9}\,\text{Nm}^2/\text{C}^2)(3.0 \times 10^{-6}\,\text{C})}{(0.30\,\text{m})^2}$$

$$= 3.0 \times 10^{5}\,\text{N/C}$$



the convention

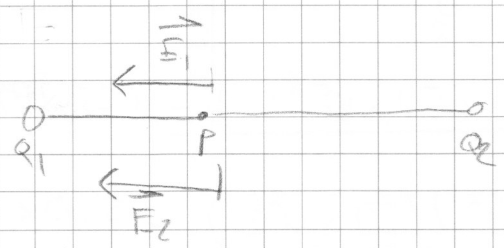
Electric field at a given point is a superposition of individual fields. Since $\vec{E}$ depends only on the source this greatly simplifies handling more complex charge distribution $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$

E at a point between two charges

a) Determine the direction and magnitude of E-field at point P

b) what is the initial acceleration of an electron at P?

(a)



$$\vec{E} = E\hat{x} = (E_1 + E_2)\hat{x}$$

$$= k\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right) = 9.0\times10^9\,N.m^2/C^2\left(\frac{25\times10^{-6}\,C}{(2.0\times10^{-2}\,m)^2} + \frac{50\times10^{-6}\,C}{(8.0\times10^{-2}\,m)^2}\right)$$

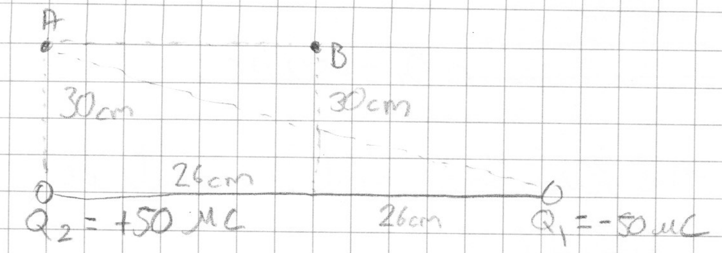
$$= 6.3\times10^8\ N/C$$

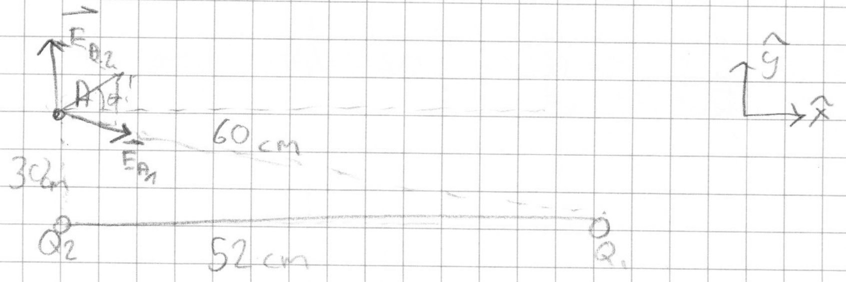
(b)

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{(1.6\times10^{-19}\,C)(6.3\times10^8\,N/C)}{9.11\times10^{-31}\,kg}$$

$$= 1.1\times10^{20}\ m/s^2$$

$\vec{E}$ above two point charges





$$E_{A_1} = \frac{(9.0 \times 10^9\ Nm^2/C^2)(50 \times 10^{-6}\ C)}{(0.60\ m)^2} = 1.25 \times 10^6\ N/C$$

$$E_{A2} = \frac{(9.0 \times 10^9\ Nm^2/C^2)(50 \times 10^{-6}\ C)}{(0.30\ m)^2} = 5.0 \times 10^6\ N/C$$

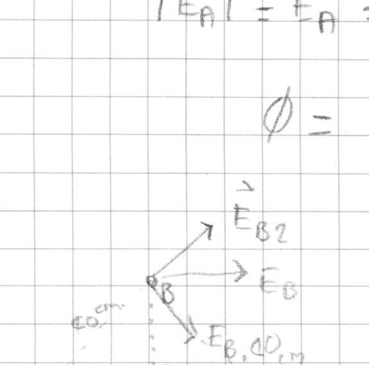
$$\vec{E}_A = E_{Ax}\hat{x} + E_{Ay}\hat{y}$$

$$E_{Ax} = E_{A_1}\cos 30 = 1.1 \times 10^6\ N/C$$

$$E_{Ay} = E_{A_2} - E_{A_1}\sin 30^\circ = 4.4 \times 10^6\ N/C$$

$$|\vec{E}_A| = E_A = \sqrt{(1.1)^2 + (4.4)^2} \times 10^6\ N/C = 4.5\ N/C$$

$$\phi = \arctan\left(E_{Ay}/E_{Ax}\right) = \arctan\left(\frac{4.4}{1.1}\right) = 76^\circ$$



By symmetry

$$E_{B1} = E_{B2} = \frac{kQ}{r^2} = \frac{(9.0 \times 10^9\ Nm^2/C^2)(50 \times 10^{-6}\ C)}{(0.40\ m)^2}$$

$$= 2.8 \times 10^6\ N/C$$

y component cancels out!

$$\vec{E}_B = E_B\hat{x} = 2E_{B1/2}\cos\theta = 2(2.8 \times 10^6\ N/C)(0.85)$$

$$= 3.6 \times 10^6\ N/C$$

Tips

1) Draw a careful free-body diagram and determine the direction of each $\vec{F}/\vec{E}$

- Unlike charges attract
- like charges repel
- $\vec{E}$ is away from + charge
- $\vec{E}$ is towards − charge
- + charge moves in the direction of $\vec{F}$
- − charge moves against the direction of $\vec{E}$

2) Apply coulomb's law for magnitudes

3) Use superposition principle and do a vector sum. Use symmetry to save time!

4) Check your results

- Do a dimensional analysis does units check?
- Are the forces reasonable?
- Do they behave reasonable at limiting cases?

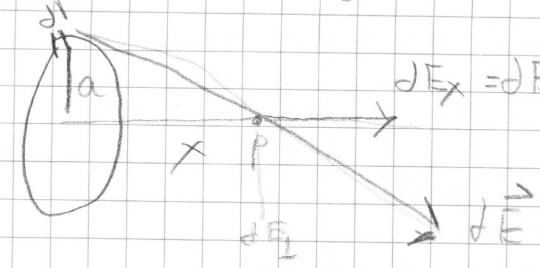
21.7 Electric field Calculations for continious charge distributions

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}$$ magnitude

$$\vec{E} = \int d\vec{E}$$ vector

......./......./........

A ring charge



1) Draw the diagram

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}$$

2) Apply Coulomb's law

$$dQ = Q\left(\frac{dl}{2\pi a}\right) = \lambda dl$$

$$\lambda = Q/2\pi a$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda dl}{r^2}$$

3) Use symmetry when adding vectorially

$dE_\perp$ cancels

$$E = E_x = \int dE_x = \int dE\cos\theta = \frac{1}{4\pi\epsilon_0}\lambda\int\frac{dl}{r^2}\cos\theta$$

$$\cos\theta = \frac{x}{(x^2+a^2)^{1/2}}$$

$$E = \frac{\lambda}{(4\pi\epsilon_0)}\frac{x}{(x^2+a^2)^{3/2}}\int_0^{2\pi a} dl = \frac{1}{4\pi\epsilon_0}\frac{\lambda x(2\pi a)}{(x^2+a^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}$$

4) Sanity check

$x \gg a \rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}$ (point charge result)

GÜLPAŞ

Note the good habits
(1) Use symmetry to reduce the complexity

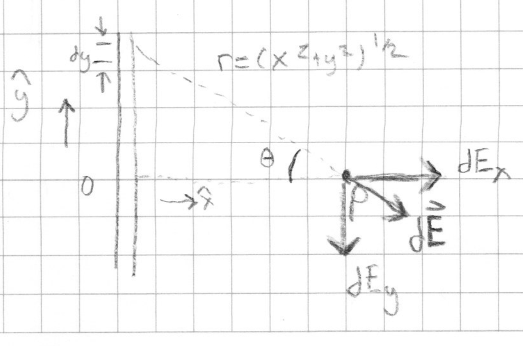
(2) express charge dQ in terms of a charge density (here linear $\lambda = Q/2\pi a$

(3) checking the answer at the limit
($r \longrightarrow \infty$)

Question ; What is the force acting on a positive charge exactly at the center of an uniformly negative charged ring
what about negative charge
- Symmetry
- positive charge : stable equilibrium
- negative charge ; unstable equilibrium

Long line charge

Determine the magnitude of the electric field at any point P a distance x from the wire, where $x \ll$ length of the wire and the charge per unit length is $\lambda (C/m)$
- Set up a coordinate system



$dQ = \lambda dy$

- $dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda dy}{(x^2+y^2)}$

$$d\vec{E} = dE_x\hat{x} + dE_y\hat{y} = dE\cos\theta\,\hat{x} + dE\sin\theta\,\hat{y}$$

- Use symmetry:

there is always a counterpart for the electric field, so, there shall be no y component, it cancels out.

In terms of integral:

$$E_y = \int_{-\pi/2}^{\pi/2} dE\sin\theta = 0$$

↳ odd function

thus

$$E = E_x = \int dE\cos\theta = \frac{\lambda}{4\pi\epsilon_0}\int\frac{\cos\theta\, dy}{x^2+y^2}$$

Cylindirical coordinates are the easiest (x parameter, integrate over y)

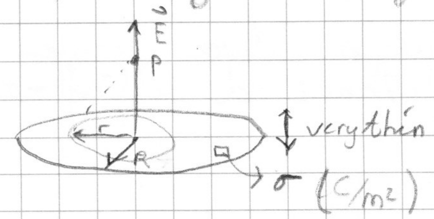
$y = x\tan\theta$

$$\frac{dy}{d\theta} = x\frac{d(\tan\theta)}{d\theta} = x\frac{d}{d\theta}\left(\sin(\theta)(\cos^{-1}(\theta))\right) = x\left((\cos(\theta)\cos^{-1}(\theta) + \sin\theta(-1)\cos^{-2}\theta(-\sin\theta)\right)$$

$$= x\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right) = x\frac{1}{\cos^2\theta} \rightarrow dy = \frac{x}{\cos^2\theta}d\theta$$

$$E = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{x}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{\lambda}{4\pi\epsilon_0 x}(\sin\theta)\Big|_{-\pi/2}^{\pi/2} = \boxed{\frac{1}{2\pi\epsilon_0}\frac{\lambda}{x}}$$

Note that the field decreases inversely as the first power uniformly charged disk



• Approach:

we can recycle the result of charged ring problem
Σ concentric rings

for one ring: $dE = \frac{1}{4\pi\epsilon_0} \frac{z\,dQ}{(z^2+r^2)^{3/2}}$ (recall the previous problem)

the ring has an area

$dr\,(2\pi r)$

$\sigma = dQ/(2\pi r\,dr)$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{z\sigma\, 2\pi r\, dr}{(z^2+r^2)^{3/2}}$$

Sum over all rings from $r=0$ to $r=R$

$$E = \frac{z\sigma}{2\epsilon_0} \int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

$u=(z^2+r^2)$ $du = 2r\,dr$ $r\,dr = \frac{1}{2} du$

$$E = \frac{z\sigma}{2\epsilon_0} \int \frac{1}{2} u^{-3/2} du = \frac{z\sigma}{2\epsilon_0} \left[\frac{1}{\cancel{2}} (-\cancel{2}) u^{-1/2} \right]$$

$$= \frac{z\sigma}{2\epsilon_0} \left[\frac{1}{\sqrt{z^2+r^2}} \right]_0^R = \boxed{\frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{(z^2+R^2)^{1/2}} \right]}$$

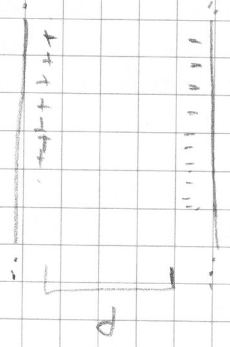
This has a very useful result at the limit $z \ll R$

$R \to \infty$ $\boxed{E = \frac{\sigma}{2\epsilon_0}}$ infinite plane

Handy: point charge $\sim 1/r^2$ line charge $\sim 1/r$ plane (no r dependence)

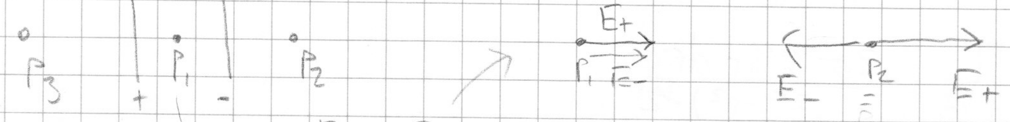
Two parallel plates:

Determine the $\vec{E}$ field between two ~~large~~ parallel plates seperated by distance d holding $-\sigma$ and σ surface charge density



use the previous solution

$$E = \frac{\sigma}{2\varepsilon_0}$$



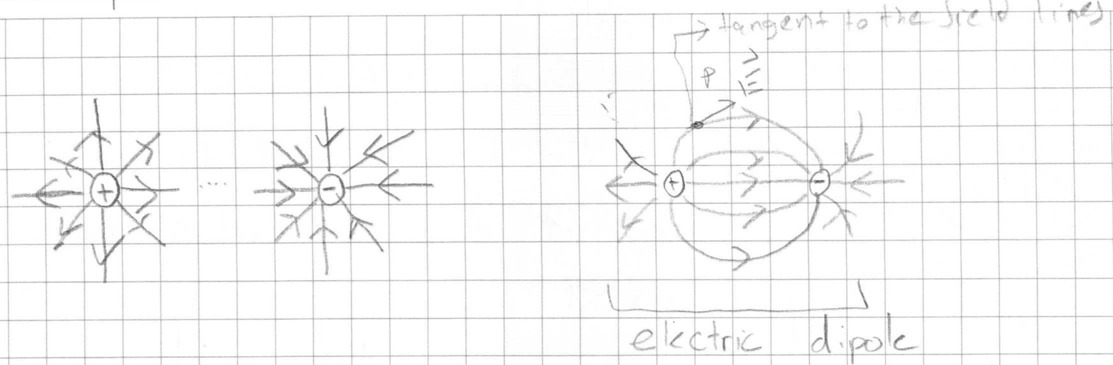
$$E = E_+ + E_- = \frac{\sigma}{\varepsilon_0}$$

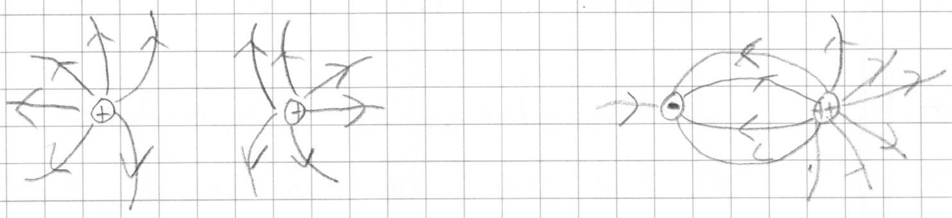
$$E = E_- - E_+ = 0$$

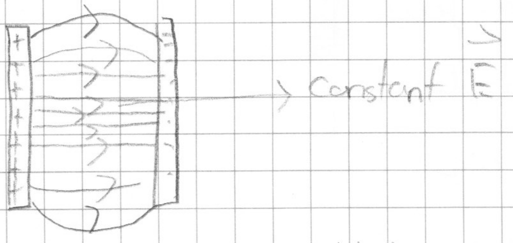
Field lines

Another tool that will become handy later ~~in your career is the vector field, especially if you start developing FEM/FET software~~

- $\vec{E}$ and $\vec{F}$ has the same direction but different magnitude
- We want to be able to know the direction of force at every point → field lines
- We can piggyback the density of field lines to carry intensity information
  → more lines → stronger field







- Field lines indicate the direction
- The density of lines is proportional to magnitude
- start on positive, end on negative
- They never cross (why: one force per test charge)
  (do the vector sum before)

Electric fields and conductors

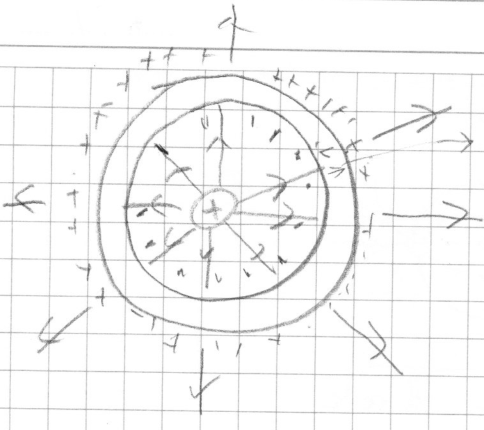
- The electrons are free to move inside a conductor

Consequences

(1) - The electric field inside the conductor is zero in the static equilibrium
(electrons move in reaction to external field until it is counteracted)

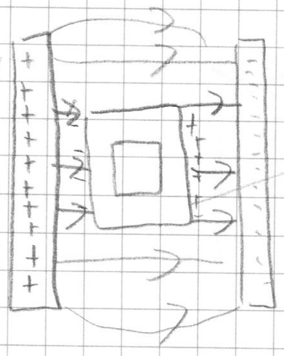
(2) - All the charge is accumulated evenly on the surface

....../....../.......



$\rightarrow E_{inside} = E_{external} + E_{induced} = 0$

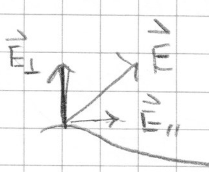
the E field outside is as if the conductor did not exist



all the charge is accumulated at the surface so inside there is no e-field (skin depth)

This is called a faraday cage

(3) The electric field is always perpendicular to the metallic surface



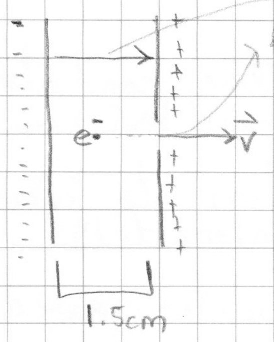
good conductor

$\vec{E}_\parallel$ is counteract along the surface

This saves lives! Airplane in a storm, Car in a storm

Motion of a charged Particle in an Electric field

$\vec{F} = q\vec{E}$



$\rightarrow E = 2.0 \times 10^4 N/C$

tiny hole

$F = ma$

$F = qE$

$a = \frac{F}{m} = \frac{qE}{m}$

$a = \frac{(1.6 \times 10^{-19} C)\ (2.0 \times 10^4 N/C)}{(9.1 \times 10^{-31} kg)} = 3.5 \times 10^{15} m/s^2$

$v_0 = 0$ $\qquad \frac{1}{2}mv^2 = F.x \Rightarrow v^2 = 2ax$

$$v = \sqrt{2ax} = \sqrt{2(3.5\times10^{15}\,m/s^2)(1.5\times10^{-2}\,m)} = 1\times10^{7}\,m/s$$

$3\times10^{9}\,m/s$

(b) The magnitude of the force on the electron

$$qE = (1.6\times10^{-19}\,C)(2.0\times10^{4}\,N/C) = 3.2\times10^{-15}\,N$$

the gravitional force

$$mg = (9.1\times10^{-31}\,kg)(9.8\,m/s^2) = 8.9\times10^{-30}\,N$$

Gravitational force can be safely ignored

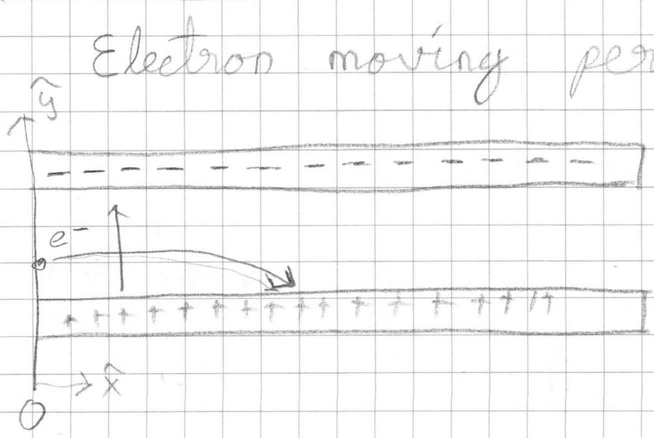
Other ignored forces Larmor's equation

$$P = \frac{2}{3}\frac{q^2 \dot{v}^2}{c^2}$$

relativistic effects

☆ The electric field due to electron does not enter to the equation, since the particle can not exert a force to itself

Electron moving perpendicular to $\vec{E}$



$F = qE$

$$a_y = \frac{F}{m} = \frac{qE}{m} = \frac{-eE}{m}$$

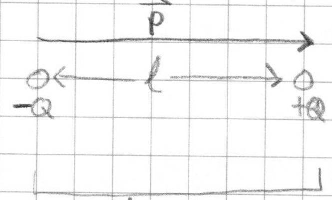
$$y = \frac{1}{2}a_y t^2 = \frac{-eE}{2m}t^2$$

$x = v_0 t$

$$\underbrace{y = \frac{-eE}{2m v_0^2}x^2}_{\text{parabola}}$$

Electric Dipoles

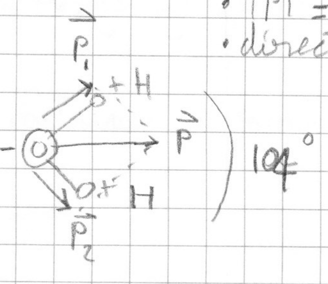
An electric $\overset{2}{\text{di}}$-pole :



<u>equal opposite</u> charges seperated by a distance ℓ

dipole moment :

- $\vec{P}$
- $|P| = Q\ell$
- direction − to + (oppossite of $\vec{E}$!)



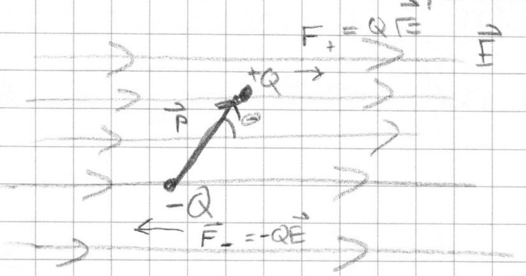
water molecule

<u>Dipole in an external field</u>

consider a dipole $p = Q\ell$ in an uniform $\vec{E}$ field

$Q \rightarrow$ fixed

$\ell \rightarrow$ fixed



no net force but a torque

around the center of the dipole

$$\tau = QE\frac{\ell}{2}\sin\theta + QE\frac{\ell}{2}\sin\theta = PE\sin\theta$$

$$\vec{\tau} = \vec{P} \times \vec{E}$$

External $\vec{E}$ tries to align $\vec{p}$ parallel to $\vec{E}$

The work done

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

in polar right hand coordinates

$$\tau = -pE \sin\theta$$

$$W = -pE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = pE\cos\theta \Big|_{\theta_1}^{\theta_2} = pE(\cos\theta_2 - \cos\theta_1)$$

positive work done by the field decreases energy

$$\Delta U = -W$$

set $U=0$ when $\vec{p}$ is perpendicular to $\vec{E}$ ($\theta = 90°$)

$$U = -W = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

when the electric field is not uniform there can be a net force on the dipole as well as the torque

Dipole in a field

The dipole moment of the water $\vec{p} = 6.1 \times 10^{-30}$ C.m if the water is placed in an uniform Electric field of 2.0×10^5 N/m

(a) what is the maximum torque?

(b) what is the potential energy when torque is maximum

(c) what is the orientation that maximises the U!

GÜLPAŞ

(a) maximum at $\theta = 90°$

$\tau = pE = (6.1 \times 10^{-30}\ C.m)(2.0 \times 10^{5}\ N/C) = 1.2 \times 10^{-24}\ N.m$

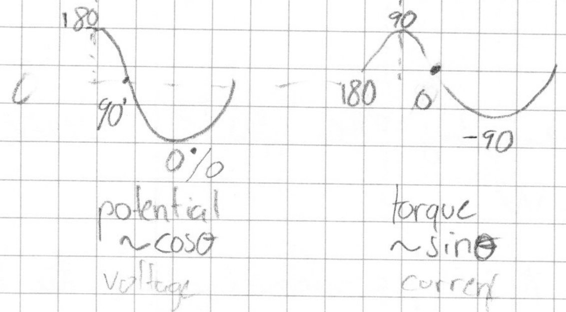
(b) Potential energy at $\theta = 90°$ is zero

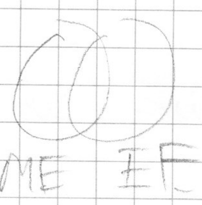
$\theta < 90$ $\quad U < 0$ $\quad$ so $\theta = 90$ is not a minimum

(c) The potential is max at $\cos\theta = -1 \rightarrow \theta = 180°$

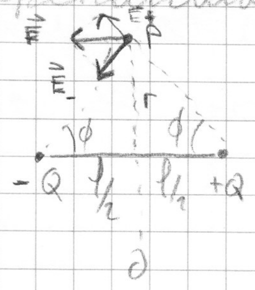
$\vec{E}$ and $\vec{P}$ are anti parallel

equilibrium position is at 0





Electric field produced by a dipole at the perpendicular bisector



$\vec{E} = \vec{E}_+ + \vec{E}_-$

$$E_+ = E_- = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2 + l^2/4}$$

y componets cancel due to symmetry

$$E = 2E_+ \cos\phi = \frac{1}{2\pi\epsilon_0}\left(\frac{Q}{r^2 + l^2/4}\right)\frac{l}{2(r^2 + l^2/4)^{1/2}} \qquad (Ql = P$$

$$= \frac{1}{4\pi\epsilon_0} \frac{P}{(r^2 + l^2/4)^{3/2}}$$

Far from the dipole $r >> l$

$$E = \frac{1}{4\pi\epsilon_0} \frac{P}{r^3} \qquad (\text{note the } r^3)$$