Gauss's Law

mathematician Karl Friedrich Gauss (1777-1855)

German

Age of Reason 1650 - 1800

30 year's war 1618-1648

harsh critisms of Nationalism and Warfare

Scientific thought

X Romanticism

End ; french revolution

why inspired by Enlightment

degraded into chaos and violence

- Enlightment as instabilit

Napoleon period (1800)

Gauss's law allows a more elegant approach for calculating the electric field due to a charge distribution

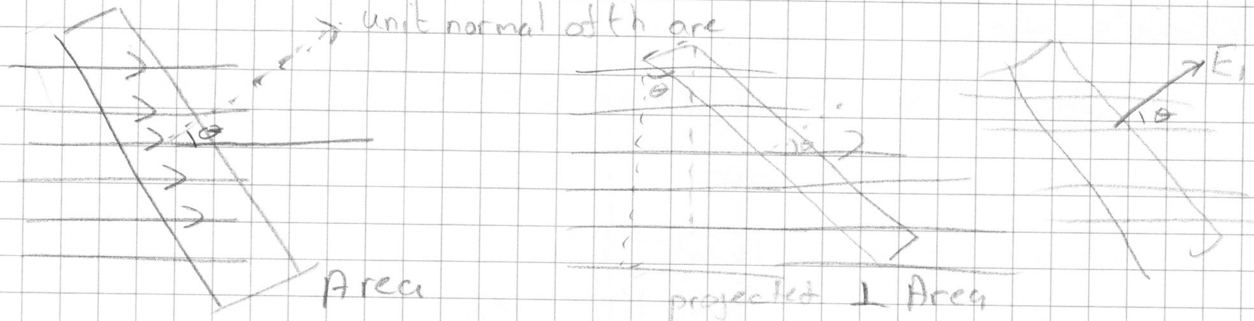
a more general relationship btw a charge and a field

Electric Flux

- Remember in the previous section, we superimposed the magnitude of the $\vec{E}$ to the density of field lines

- The formal method for this is ;

....../....../.......



The flux

$$\phi_E = EA\cos\theta$$

$$= E_\perp A = EA_\perp = EA\cos\theta$$

The area can be represented by $\vec{A}$ where A is the area, and the direction is the unit normal

$$\phi_E = \vec{E}\cdot\vec{A}$$

The magnitude of the field is conveyed by the flux $E \propto N/A_\perp$

$$N \propto EA_\perp = \phi_E$$

Example

If the rectangle above is 10 cm x 20 cm and $\vec{E}$ is uniform 200 N/C and $\theta = 30$

$$\phi_E = (200\,N/C)(0.10\,m \times 0.20\,m)\cos 30^\circ$$

$$= 3.5\ Nm^2/C$$

Chapter 22 P2 V1

19/02/2019

when the $\vec{E}$ is not uniform and the surface is not flat:

divide and Conquer

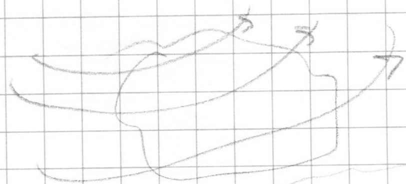
$$A \rightarrow \Delta A_1 + \Delta A_2 + \cdots$$

(1) ΔA_n are so small that they can be considered flat

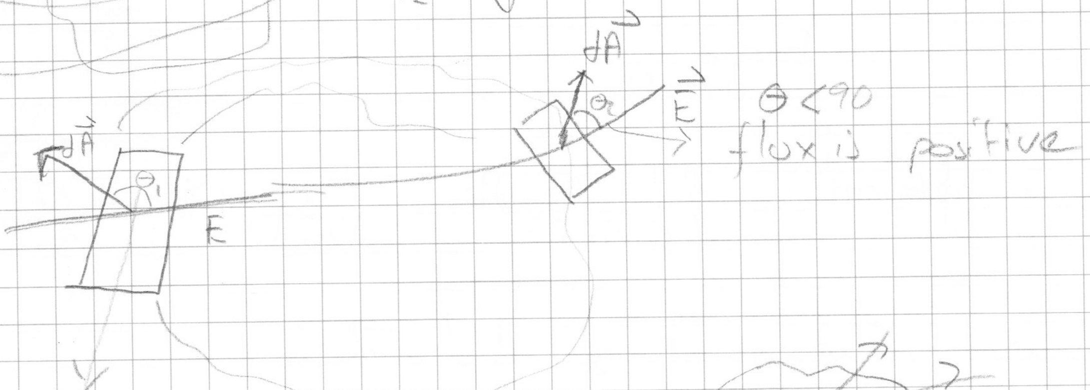
(2) When ΔA_n is small enough $\vec{E}$ varies so little that it can be considered uniform

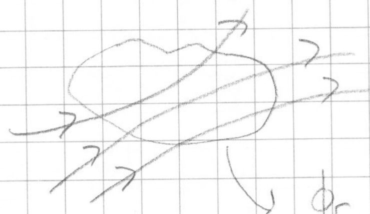
$$\phi_E \simeq \sum_{i=1}^{N} \vec{E}_i \cdot \Delta\vec{A}_i \qquad \underbrace{\phi_E = \int \vec{E} \cdot d\vec{A}}_{\text{exact}}$$

Gauss's Law involves closed surface i.e a surface that completely encloses a Volume



$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$





$$\phi_E = \oint \vec{E}\, d\vec{A} = 0$$

(no sources or sinks)

G

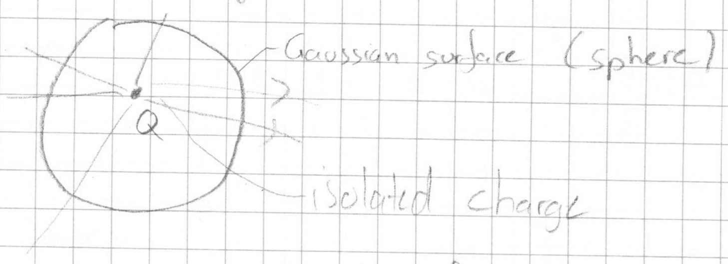
GÜLPAŞ

Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

- The distribution of Electric charge does not matter as long as it is enclosed by the surface. only the net charge matters

Relation of Gauss's law and Coulomb's law (



$$\left.\begin{aligned} &\oint \vec{E} \cdot d\vec{A} = \oint E\,dA = E \oint dA = E(4\pi r^2) \\ &\frac{Q}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) \end{aligned}\right] \text{Gauss's} \xrightarrow{} \text{Coulomb's}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\left.\oint \vec{E}\, d\vec{A} = \oint \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}\, dA = \frac{Q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{Q}{\epsilon_0}\right] \text{Coulomb's} \xrightarrow{} \text{Gauss's}$$

what about a charge distribution

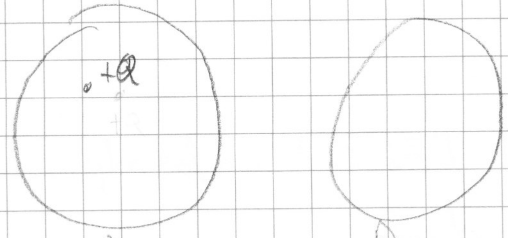
$$\vec{E} = \sum \vec{E_i}$$

$$\oint \vec{E_i} \cdot d\vec{A} = \frac{Q_i}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \oint \sum \vec{E_i}\, d\vec{A} = \sum \oint \vec{E_i} \cdot d\vec{A} = \sum \frac{Q_i}{\epsilon_0} = \frac{Q_{encl}}{\epsilon_0}$$

Gauss's law is more handy than Coulomb's law since it also holds for electric fields due to changing $\vec{B}$ fields
valid for all kinds of E fields

GÜLPAŞ



A_1 A_2

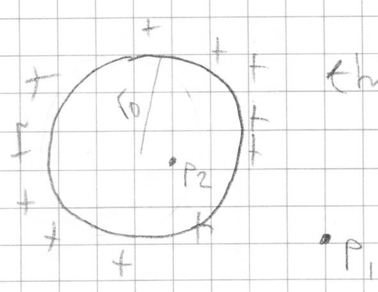
what is the net flux through A_1 and A_2?

$= Q/\epsilon_0$ $= 0$

Application of Gauss's Law

- Choose the Gaussian surface very carefully
- Try to use symmetry to your benefit
- Try to identify the easiest solution

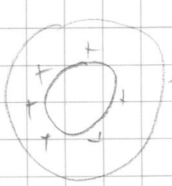
Example



thin metallic spherical shell

- metallic → charge must be distributed evenly

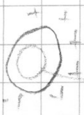
even charge → even field (must not depend on θ)



→ gaussian surface where $\cos\theta = 1$

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = Q/\epsilon_0$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$



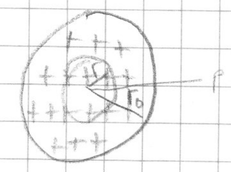
→ gaussian surface

$$\oint \vec{E}\, d\vec{A} = E(4\pi r^2) = 0$$

$$E = 0$$

Solid sphere of charge

An electric charge Q is distributed uniformly throughout a non-conducting sphere



uniform distribution $\rightarrow$ charge depends on r

out: $\oint \vec{E} \cdot d\vec{A} = E\underbrace{(4\pi r^2)}_{\text{surface of sphere}} = \frac{Q}{\epsilon_0}$

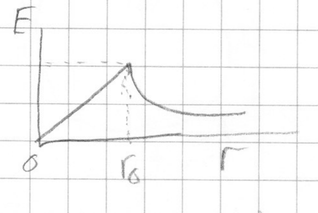
$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

inside: $\oint E \cdot d\vec{A} = E(4\pi r^2)$

$$Q_{encl} = \left(\frac{4}{3}\pi r^3 \rho_0\right) = \frac{4\pi r^3 Q}{4\pi r_0^3} = \frac{r^3}{r_0^3} Q$$

$$E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0} = \frac{r^3}{r_0^3}\frac{Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^3} r$$



would've been quite difficult using Coulomb's law

- properties of E field, symmetries

$E \propto Q$

Non-uniformly charged solid sphere

$\rho_E = \alpha r^2$ total charge: Q

let's get rid of α

$$Q = \int \rho_E \, dV = \int_0^{r_0} (\alpha r^2)(4\pi r^2 dr) = 4\pi\alpha \int_0^{r_0} r^4 dr$$

$$= \frac{4\pi\alpha}{5} r_0^5$$

Chapter 22 P4 V1 — 23.02.2014

$$Q_{encl} = \int_0^r \rho_E \, dV = \int_0^r (\alpha r^2)\, 4\pi r^2 dr$$

$$= \int_0^r \left(\frac{5Q}{4\pi r_0^5} r^2\right) 4\pi r^2 dr = Q \frac{r^5}{r_0^5}$$

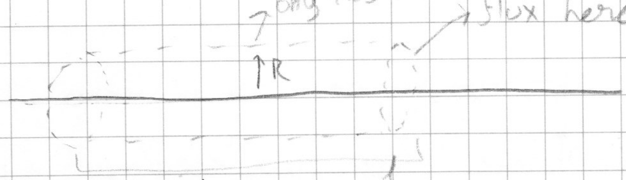
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$(E)(4\pi r^2) = \frac{Q}{\epsilon_0} \frac{r^5}{r_0^5}$$

$$E = \frac{Q r^3}{4\pi \epsilon_0 r_0^5}$$

Long uniform line of charge

flux here is 0 $(\cos 90 = 0)$

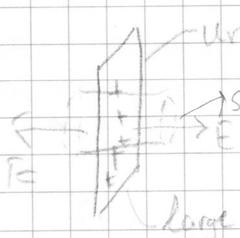


$$\oint \vec{E} \cdot dA = E(2\pi R l) = \frac{Q_{encl}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{R}$$

Infinite plane of charge

uniform charge density σ (dQ/dA)



small closed cylinder

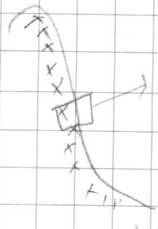
large and thin

$\vec{E}$ near plane

side walls $\cancel{\oint} \vec{E} \cdot d\vec{A} = 0$

$$\oint \vec{E} \cdot d\vec{A} = 2EA = \frac{Q_{encl}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \sigma / 2\epsilon_0$$



→ small gaussian surface where surface is essentially flat

→ conductor

Show that $\vec{E}$ near the surface of a conductor is $E = \frac{\sigma}{\epsilon_0}$
(just asks the magnitude)

Same approach as above but the $\vec{E}$ inside the conductor is zero

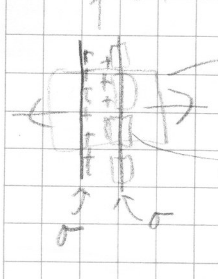
$$\oint \vec{E} \cdot d\vec{A} = EA = \frac{Q_{encl}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

So $E = \frac{\sigma}{\epsilon_0}$

→ notice this is twice as previous

metal

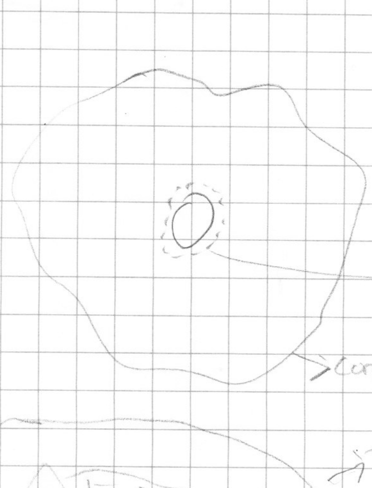
important result for future when you learn about mirrors



→ Previous result

→ our result

σ σ $\sigma' = 2\sigma$

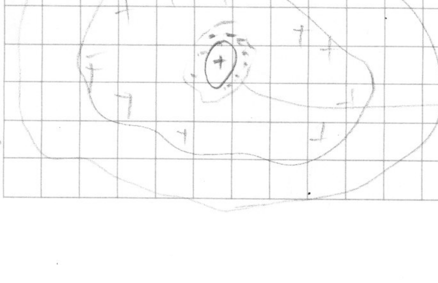


→ gaussian just below internal surface

→ conductor

$\oint \vec{E} \cdot d\vec{A} = 0$ so charge is 0

0 (conductor)



→ field is non-zero

→ Since field is still zero
there must be equal amount of − to +