Describing Motion:
Kinematics in one dimension

It was not until the 16th and 17th century that our modern understanding of motion was established

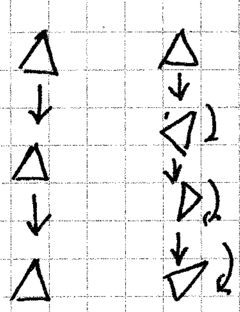
Many individuals contributed
- Galileo Galilei (1564) – (1642)
- Isaac Newton (1642) – (1727)

Mechanics (the study of the motion of objects)
- Kinematics (how objects move)
- Dynamics (force, why objects move as they do)

Large objects that are not massive, speeds not approaching speed of light → classical mechanics
If the present state is known we know the future and past (determinism, reversibility)

((Classical Mechanics)

For now we will focus on objects that move without rotating



translational → translation + rotation

The concept of point particle

A mathematical point with no spatial extent (0D)

this allows us to seperate translational motion and ratational motion (then recombine later)

1) Reference frame

What is your speed now?

Ankara Lat: 40°
Radius : 6370 km
Circumference ~ 40.000 km
23 hours 56 minutes 4.09 seconds per turn

~ 1668 km/hr w.r.to earth's center

~ 108 000 km/hr w.r.to Sun

~ 792 000 km/hr w.r.to Milky way's center

~ 1 404 000 km/hr w.r.to CBR (towards LeO)

(Light speed ~ 10^9 km/hr)

The reference frame must be specified whenever there might be confusion.

Three-dimensional space : three points are required to determine the position of an element

(In physics and in Mathematics a sequence of n-numbers can be understood as a location in n-dimensional space)

Three dimensional Euclidean space ($\mathbb{R}^3$)

Euclid of Alexandria ~ 300 BC

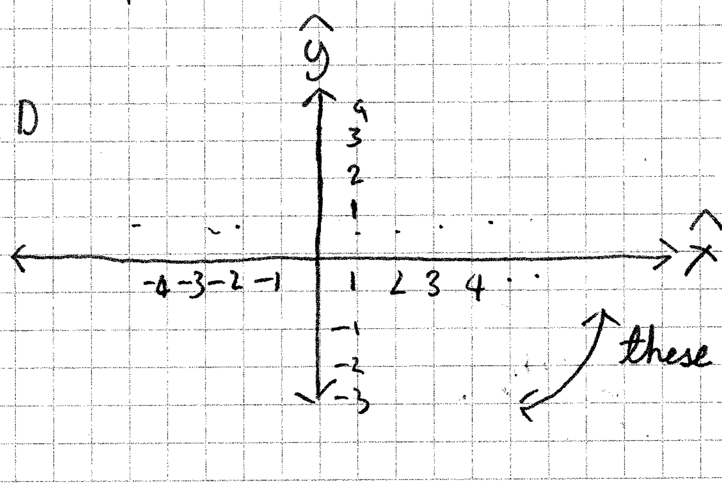
Euclidean Space is an abstraction of our physical space, describing relations with as few elements as possible

axioms ← elements → theorems

Euclidean geometry → Algebra connection Rene Descartes 1637

1D i.e circle $x^2+y^2=4$ x^{2}

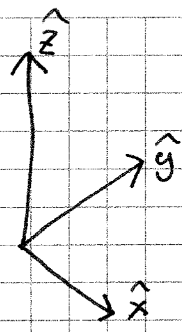
2D



→ direction

these are completely independent

3D



Convention: Right Handed System

Thumb +x

Fingers: +y

palm: + z

... (to n dimensions)

Displacement vs. Distance

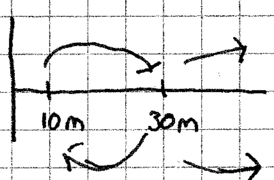
Displacement: Change in position

- How far
- In which direction

] Vector

Δ (Delta)(change in ...)

Δx : Change in x coordinate



$\Delta x = x_2 - x_1 = 30m - 10m = +20m$

direction

$\Delta x = x_2 - x_1 = 10m - 30m = -20m$

vector

general displacement (next chapter) $\vec{r} = \underbrace{\Delta x \hat{x}}_{\Delta \vec{x}} + \underbrace{\Delta y \hat{y}}_{\Delta \vec{y}} + \underbrace{\Delta z \hat{z}}_{\Delta \vec{z}}$

Average velocity

Speed: how far an object travels in a given time interval

per

Campus speed limit: 30 km/h ← given time

how far

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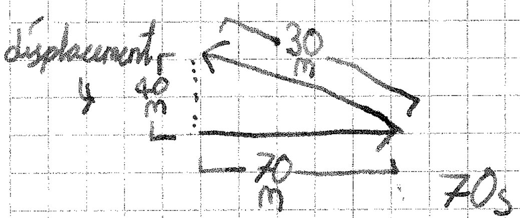
$$\text{average speed} = \frac{\text{distance travelled}}{\text{time elapsed}}$$

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}}$$

is average speed equal to magnitude of average velocity ?



average speed $= \frac{100\,m}{70\,s} = 1.4\,m/s$

$|\text{average velocity}| = \frac{40\,m}{70\,s} = 0.57\,m/s$

magnitude of

So, be extra careful when dealing with averages, there is loss of information

elapsed time : $\Delta t = t_2 - t_1$

elapsed displacement : $\Delta x = x_2 - x_1$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1}$$

average

Instantaneous velocity

: the average velocity over an infi[illegible]simally short time interval

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{d\vec{x}}{dt}$$

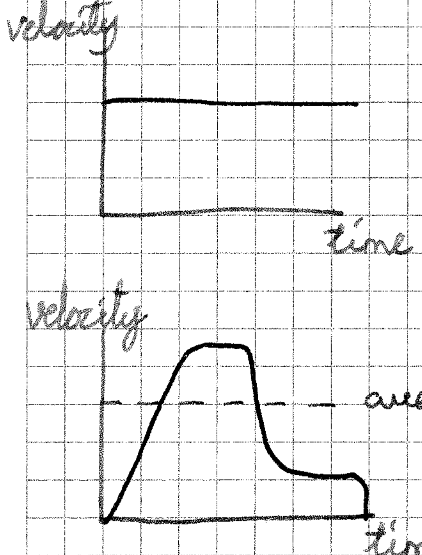
$\Delta \to d$ (for infitesimal)

"velocity" refers to instantaneous velocity

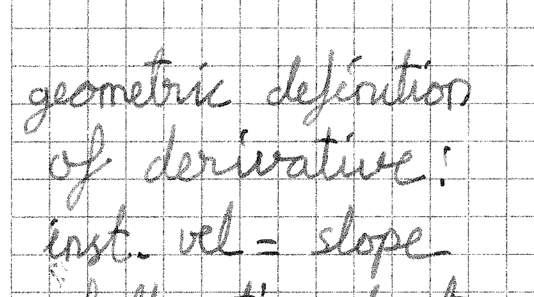
average velocity
↑ explicitly mentioned

(instantaneous speed = magnitude of instantaneous velocity)

because magnitude of displacement = distance travelled when dealing with infinitesimal quantities

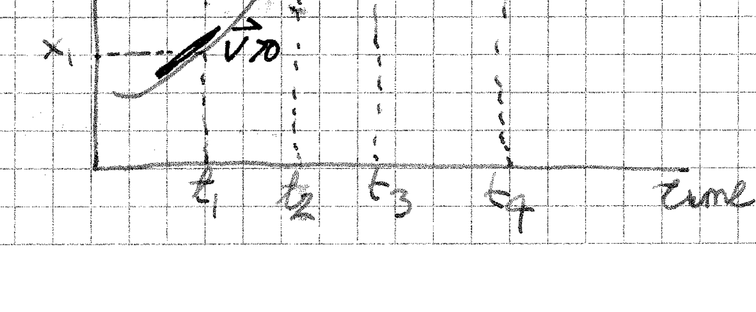


uniform (constant) velocity
instantaneous vel. = average vel. at all times!



more typically, instantaneous and average velocities differ

geometric definition of derivative:
inst. vel = slope of the tangent to the curve at that point



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Example

$x = At^2 + B \quad A = 2.10\ m/s^2 \quad B = 2.80\ m$

a) displacement from $t_1 = 3.00s$ to $t_2 = 5.00s$

b) av. velocity

c) inst vel. at $t = 5.00s$

a) at t_1 : $x_1 = (2.10\ m/s^2)(3.00s)^2 + 2.80m$

$= 21.7m$

t_2 : $x_2 = (2.10\ m/s^2)(5.00s)^2 + 2.80\ m$

$= 55.3\ m$

displacement $= 55.3m - 21.7m = +33.6\ m$

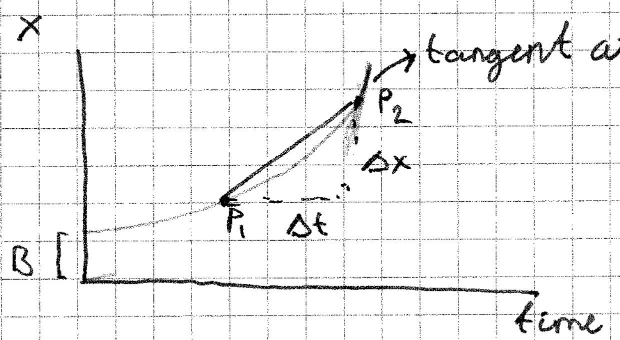
(b) the magnitude of average velocity

$$\vec{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{+33.6m}{2.00s} = 16.8m/s$$

(c)

$$\vec{v} = \left.\frac{dx}{dt}\right|_{t_2 = 5.00s} = \left.\frac{d}{dt}(At^2 + B)\right|_{t_2 = 5.00s} = \left.2At\right|_{t = 5.00s} = 2(2.1\ m/s^2)(5s)$$

$= 21.0\ m/s$



Acceleration

change in velocity : accelleration

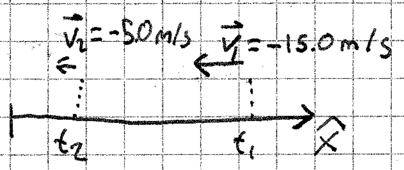
average accelleration = $\dfrac{\text{change of velocity}}{\text{time elapsed}}$

$$\bar{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

i.e: $\bar{a} = \dfrac{25 m/s - 0 m/s}{5.0\ s} = 5.0\ \dfrac{m/s}{s} = 5 m/s^2$

Unit	Measurable	dimension
m	: distance, position	
m/s	: Velocity (how fast the position changes)	
$(m/s)/s = m/s^2$	: acceleration (how fast the ~~accelera~~ velocity changes)	

Deceleration



the object is slowing down (Decelerating

Is the acceleration negative or positive?

positive!

Instantaneous Acceleration

Instantenous acceleration($\vec{a}$) = limiting value of the average acceleration as we let $\Delta t \to 0$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$ → derivative of $\vec{v}$ w.r.to t

example

$$x(t) = (2.10\ m/s^2)t^2 + (2.8m)$$

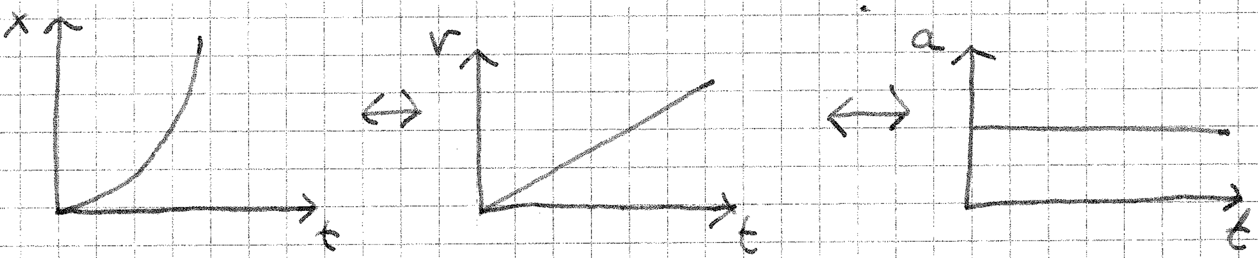
$$\vec{v} = \frac{dx}{dt} = \frac{d}{dt}\left[(2.1\ m/s^2)t^2 + (2.8m)\right] = (4.2\ m/s^2)t$$

$$\bar{a} = \frac{\Delta V}{\Delta t} = \frac{21.0\ m/s - 12.6\ m/s}{5.00s - 3.00s} = 4.2\ m/s^2$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(4.2m/s^2)t = 4.2\ m/s^2$$

note:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}$$ → Second derivative



Motion at Constant acceleration

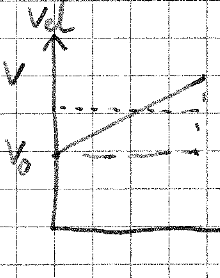
velocity: set $t_1 = 0$ and $t_2 = t$

$$\vec{a} = \bar{a} = \frac{\vec{v} - \vec{v}_0}{t_2 - t_1} = \frac{\vec{v} - \vec{v}_0}{t}$$

(1) $V = V_0 + at$

position

one can use average velocity to calculate position at any given time



$\bar{v} = \frac{v_0 + v}{2}$ or average $= \frac{1}{b-a} \int_a^b f(x)\,dx$

$$\frac{1}{t} \int_0^t v_0 + at'\,dt' = \frac{1}{t}\left(v_0 t + \frac{1}{2} a t^2\right)$$

$$= v_0 + \frac{1}{2} a t = v_0 + \frac{1}{2} v - \frac{1}{2} v_0$$

$$= \frac{v_0 + v}{2}$$

then:

$$x = x_0 + \bar{v} t \quad \text{(using av. vel.)}$$

$$= x_0 + \left(\frac{v_0 + v}{2}\right) t = x_0 + \left(\frac{v_0 + v_0 + at}{2}\right) t$$

(2) $$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

unknown time

$$x = x_0 + \vec{v} t = x_0 + \left(\frac{v + v_0}{2}\right) t$$

$$\rightarrow \quad t = \frac{v - v_0}{a}$$

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}$$

(3) $$v^2 = v_0^2 + 2a(x - x_0)$$

Solving Problems:

Physics is not a collection of equations to be memorized!

→ first principles thinking!!

1.) Read carefully
2.) What object, What interval?
3.) Draw a diagram, put down coordinate system
4.) What do you know, what do you want to know?
5.) What first principles can be used?
6.) Apply shortcuts after validating them
7.) Calculate
8.) Does the result make sense?
9) Do the units hold?

Freely Falling objects

Does an heavier object fall faster than lighter one?

Gallileo Gallilei: The father of scientific method (and his gift was house arrest for the rest of his life)

(at a given location on the earth and in the absence of air resistance, all objects fall with the same constant accelleration) at the time of Gallileo it was not possible to create vacuum. He generalised this using several experiments

paper vs stone (change shape of paper)
same shape heavy objects
heavy objects from different heights

Empiricism: emprical evidence
i.e. any complex belief we hold can be true, only if parts that make it are true

an idea that is formed from true first principles will itself be true

acceleration due to gravity $g = 9.80 \ m/s^2$
(actually this varies throughout the earth a little that is why satellites need frequent course corrections)

Air resistence leads to some effects such as Terminal velocity, but in the domain of speeds and objects we consider, most air resistance effects can be ignored

Accelleration due to gravity is a vector as in any acceleration, pointing towards center of earth

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Falling from a tower

A ball is dropped (i.e. $v_0 = 0$) from a tower
70.0m high
How far will it have fallen after
$t_1 = 1s \quad t_2 = 2s \quad t_3 = 3s$
Ignore air resistance

coordinate system

$\downarrow +y$

known $\quad a = +9.80\ m/s^2\ \hat{y}$

$v_0 = 0$

$y_0 = 0$

$y_1 = y(1s) \quad y_2 = y(2s) \quad y_3 = y(3s)$

$$y_1 = v_0 t_1 + \frac{1}{2} a t_1^2 = 0 + \frac{1}{2} a t_1^2 = \frac{1}{2}(9.8\ m/s^2)(1.00 s)^2 = 4.9 m$$

$y_2 = 19.6 m$ (

$y_3 = 44.1 m$

Thrown down from a tower

A ball is thrown downwards (i.e $v_0 = 3.00\ m/s$)
rest is the same
a) position b) speed

$$y_1 = (3.00\ m/s)(1.00 s) + \frac{1}{2}(9.8\ m/s^2)(1.00 s)^2 = 7.9 m$$

$y_2 = 25.6 m \quad y_3 = 53.1 m$

......./......./.......

b)

$v = v_0 + at$

$v_1 = 3.00 m/s + (9.80 m/s^2)(1.00 s) = 12.8 m/s$

$v_2 = 3.00 m/s + (9.80 m/s^2)(2.00 s) = 22.6 m/s$

Ball thrown upward

A ball is thrown upward with an initial velocity of 15.0 m/s

a) How high does it go?

b) How long does it stay in air before it comes back to same point it was launched?

- Maximum height : when $v=0$

$$v^2 = v_0^2 + 2a(y-0) \rightarrow y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0 m/s)^2}{2(-9.8 m/s^2)} = 11.5 m$$

for second part:

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

$$0 = 0 + (15.0 m/s)t + \frac{1}{2}(-9.80 m/s^2)t^2$$

$$(15.0 m/s - 4.90 m/s^2 t)t = 0$$

$t = 0 s$

launch

$t = \frac{15.0 m/s}{4.9 m/s^2} = 3.06 s$

return

Is acceleration and velocity are always in same direction?

No!

Is accellaration zero at top?

No!

c) When does the ball reach topmost position of its trajectory

d) What is the velocity of the ball when it returns back?

c) $V = V_0 + at$

$\underset{0}{\uparrow}$

$t = \frac{-V_0}{a} = -\frac{15.0\,m/s}{-9.80\,m/s^2} = 1.53\,s$ (half the time)

d) $V = V_0 + at = 15.0\,m/s - (9.8\,m/s^2)(3.06\,s) = -15.0\,m/s$

(opposite direction, same magnitude)

e) at what time the ball passes $y = 8.00\,m$

$y = y_0 + V_0 t + \frac{1}{2} a t^2$

$8.00m = 0 + (15.0\,m/s)t + \frac{1}{2}(-9.8\,m/s^2)t^2$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad at^2 + bt + c = 0$

$t = \frac{15.0\,m/s \pm \sqrt{(15.0\,m/s)^2 - (4(4.90\,m/s^2)(8.0m))}}{2.(4.90\,m/s^2)}$

$t = 0.69\,s \qquad t = 2.37\,s$

are both valid?

yes!