Solution Sheet

Phys 213 First Midterm Examination 12/11/2022 16:00-18:00

Proctor's remark:				Initials	
Full Name		Student ID		Signature	

Problem 1 of 3 (30 pts) Answers without solution steps clearly shown will not be given any credit.

Estimate the maximum allowable pressure in a 32-cm-long cathode ray tube if 98% of all electrons must hit the screen without first striking an air molecule. Assume the electrons move much faster than the air molecules, point sized objects, and move at the same speed. Also assume that the dilute gas in the tube behaves like an ideal gas.

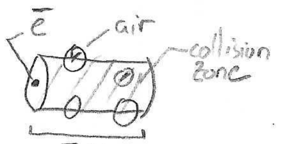
(a) (5 pts) We need to begin translating the above requirement to a mean free path l_m in this question. Mean free path can be thought as the distance %50 of the electrons should have a collision. What is the mean free path that satisfies the given criteria?

%2 → 32cm

%50 → $\boxed{8.0\,m}$

(b) (15 pts) In the kinetic theory of gases chapter, we derived the mean free path as $l_m = \frac{1}{4\pi\sqrt{2}r^2(N/V)}$ where r is the radius of the molecule. However, this derivation was done for collisions between the *same* molecules moving at *similar* speeds. Considering the electrons move much faster than air, and point like objects compared to air molecules, how will this formula change?

	before	after
Collision criteria	$2r_{air}$	r_{air}
relative speed	$\sqrt{2}\,\bar{v}_{air\text{-}air}$	$\bar{v}_{electrons}$



distance: $\bar{v}\Delta t$

Volume: $\pi r^2 \bar{v}\Delta t$

Number of air molecules inside: $\frac{N}{V}\pi r^2 \bar{v}\Delta t$

$$l_m = \frac{\text{distance travelled}}{\text{Number of collisions}} = \frac{\cancel{\bar{v}}\,\cancel{\Delta t}}{\frac{N}{V}\pi r^2 \cancel{\bar{v}}\,\cancel{\Delta t}}$$

$$\boxed{l_m = \frac{1}{\frac{N}{V}\pi r^2}}$$

(c) (10 pts) Use ideal gas law $PV = NkT$ to calculate the pressure requirement from the mean free path.

$$PV = NkT \rightarrow \frac{N}{V} = \frac{P}{kT}$$

$$l_m = \frac{1}{\frac{N}{V}\pi r^2} = \frac{kT}{\pi r^2 P}$$

$$\boxed{P = \frac{kT}{\pi r^2 l_m}}$$

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Problem 2 of 3 (30 pts) Answers without solution steps clearly shown will not be given any credit.

Derive the relation between the pressure P and the volume V of an ideal gas that is allowed to slowly expand adiabatically.

(a) (5 pts) Define heat flow (Q) in and out of the system during a quasi-static (slow) adiabatic expansion process.

$\boxed{\Delta Q = 0}$ or $\boxed{Q = 0}$

(b) (5 pts) Take the differential of the ideal gas law, $PV = nRT$ allowing P,V and T to vary

$$\boxed{PdV + VdP = nRdT}$$

(c) (10 pts) Use first law of thermodynamics $\Delta E_{\text{int}} = Q - W$, kinetic theory for internal energy of an ideal gas $\Delta E_{\text{int}} = nC_V \Delta T$, to show $(C_V + R)PdV + C_V V dP = 0$

$$dE_{int} = nC_V dT = dQ - dW = -dW = -PdV$$

$$\rightarrow nC_V dT + PdV = 0$$

use (b)

$$R.\ nC_V\left(\frac{PdV + VdP}{nR}\right) + PdV = 0\ .R$$

$$\boxed{(C_V + R)PdV + C_V VdP = 0}$$

(d) (10 pts) Show that $PV^\gamma =$ constant for a quasi-static adiabatic process in an ideal gas where $\gamma = C_P/C_V$ (tip: what is $C_V + R$ for an ideal gas)?

$C_V + R = C_p$ (ideal gas)

$$C_p PdV + C_V VdP = 0$$

γ

$$\left(\frac{C_P}{C_V}\right)PdV + VdP = 0$$

$$\int \frac{dP}{P} + \gamma\frac{dV}{V} = \int 0$$

$$\ln P + \gamma \ln V = \text{constant}$$

$$\ln P + \ln V^\gamma = \text{constant}$$

$$\ln PV^\gamma = \text{constant}$$

$$\boxed{PV^\gamma = \text{constant}}$$

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Problem 3 of 3 (40 pts) Answers without solution steps clearly shown will not be given any credit.

The specific heat of water is C_w and its density is ρ_w. Calculate the maximum useful work that can be extracted, using the source as V volume of hot water at T_H and the sink as a cold lake of temperature T_C .

(a) (20 pts) Using the maximum efficiency possible in an engine and the definition of efficiency . (tip1: what is the relationship between the heat transferred from the hot water dQ, and change in the internal energy of the hot water? See previous question if you don't remember. Tip2: $e = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H}$)

Maximum efficiency → Carnot efficiency $\frac{Q_L}{Q_H} = \frac{T_L}{T_H} = \frac{T_C}{T_H}$

$e = \frac{dW}{dQ} = 1 - \frac{T_L}{T_H}$ where dQ is the heat transfer to the lake. $dQ = -MC_w dT$

$$W = \int_{T_H}^{T_C} dW = -\int_{T_H}^{T_C} MC_w\left(1 - \frac{T_C}{T}\right)dT = -MC_w\left(T - T_C \ln T\right)\Big|_{T_H}^{T_C}$$

$$= MC_w\left[T_H - T_C - T_C \ln\left(\frac{T_H}{T_C}\right)\right]$$

$$\boxed{= \rho_w V C_w\left[T_H - T_C - T_C \ln\left(\frac{T_H}{T_C}\right)\right]}$$

(b) (20 pts) Using entropy. (tip: $(M + M_L)C_W \ln(T_C + \delta) \approx (M + M_L)C_W \ln T_C + \frac{(M+M_L)C_W\delta}{T_C}$ where M is the mass of the hot water and M_L is the mass of the lake)

The maximum heat can be extracted when entropy remains the same

$$S = MC_w \ln T_H + M_L C_w \ln T_C = (M+M_L)C_w \ln T$$

$T = T_c + \delta$ (lake)

$$(M+M_L)C_w \ln(T_c+\delta) \approx (M+M_L)C_w \ln T_c + \frac{(M+M_L)C_w\delta}{T_c}$$

Put back

$$\delta = \frac{M}{M+M_L} T_c \ln \frac{T_H}{T_c}$$

$$W = E_i - E_f = MC_w T_H + M_L C_w T_c - (M+M_L)C_w(T_c+\delta)$$

$$= MC_w T_H + M_L C_w T_c - (M+M_L)C_w\left(T_c + \frac{M}{M+M_L} T_c \ln\frac{T_H}{T_c}\right)$$

$$= MC\left(T_H - T_c - T_c \ln\frac{T_H}{T_c}\right) = \boxed{\rho_w V C_w\left(T_H - T_c - T_c \ln\frac{T_H}{T_c}\right)}$$