Phys 213 Second Midterm Examination   17/12/2022   16:00-18:00

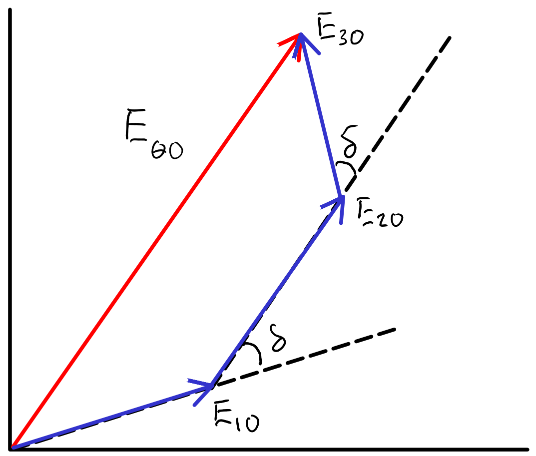
Proctor's remark:				Initials	
Full Name		Student ID		Signature	

Problem 1 of 3 (30 pts)   Answers without solution steps clearly shown will not be given any credit.

a) (20 pts) Consider three equally spaced and equal-intensity coherent sources of light (such as adding a third slit to the two slits). Use the phasor method to obtain the intensity as a function of the phase difference δ, seperation between the coherent sources d and angular position θ

$$\delta = \frac{2\pi}{\lambda} d \sin\theta$$

equal magnitude: $E_{10} = E_{20} = E_{30}$



$$E_{\theta 0} = E_{10}\cos\delta + E_{20} + E_{30}\cos\delta$$
$$= E_{10}(1+2\cos\delta)$$

$$\frac{I_\theta}{I_0} = \frac{\overline{E_{\theta 0}}^2}{E_{\delta=0}^2} = \frac{\left(E_{10}(1+2\cos\delta)\right)^2}{\left(E_{10}(1+2\cos 0)\right)^2}$$

$$\boxed{= \frac{(1+2\cos\delta)^2}{9}}$$

$$\delta = \frac{2\pi}{\lambda} d \sin\theta$$

b) (10 pts) Determine the positions of maxima and minima.

$$\cos\delta_{max} = 1$$

$$\rightarrow \delta_{max} = 2m\pi = \frac{2\pi}{\lambda} d \sin\theta_{max}$$

$$\boxed{\sin\theta_{max} = \frac{m\lambda}{d}, \; m=0,1,2,\ldots\ldots}$$

$$1+2\cos\delta_{min} = 0$$

$$\delta_{min} = \cos^{-1}(-1/2) = \begin{cases} \frac{2}{3}\pi + 2m\pi = 2\pi(m+1/3) \\ \frac{4}{3}\pi + 2m\pi = 2\pi(m+\frac{2}{3}) \end{cases}$$

$m=0,1,2,\ldots$

$$\delta_{min} = 2\pi(m+\tfrac{1}{3}k) = \frac{2\pi}{\lambda} d \sin\theta_{min} \quad k=1,2 \quad m=0,1,2,\ldots$$

$$\sin\theta_{min} = \lambda/d\,(m+\tfrac{1}{3}k) \quad k=1,2 \quad m=0,1,2,\ldots$$

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Problem 2 of 3 (30 pts) Answers without solution steps clearly shown will not be given any credit.

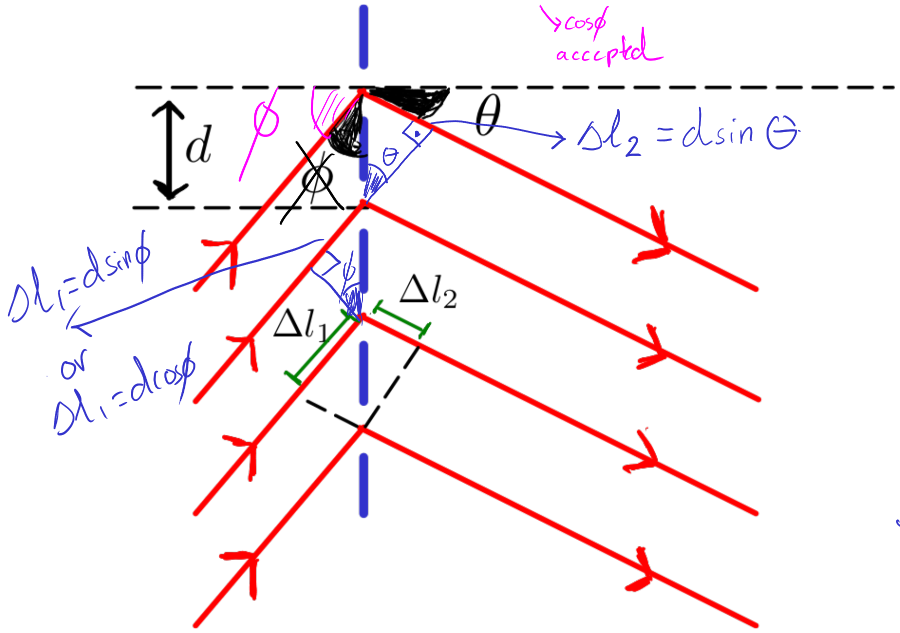
Monochromatic light falls on a transmission diffraction grating at an angle ϕ to the normal. Show that

$$\sin\theta = \frac{m\lambda}{d} \quad m = 0, 1, 2, \ldots$$

for diffraction maxima must be replaced by

$$d(\sin\phi + \sin\theta) = \pm m\lambda \quad m = 0, 1, 2, \ldots$$

cosφ accepted



$$\Delta l = \Delta l_1 + \Delta l_2$$
$$= d(\sin\phi + \sin\theta)$$

Maxima at $\Delta l = \pm m\lambda$

$$d(\sin\phi + \sin\theta) = \pm m\lambda$$

cosφ is accepted

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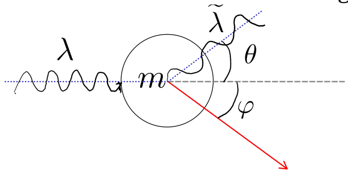
Problem 3 of 3 (40 pts) Answers without solution steps clearly shown will not be given any credit.

In the Compton effect, a γ-ray photon of wavelength λ strikes a free, but initially stationary, electron of mass m. The photon is scattered an angle θ and its scattered wavelength is $\widetilde{\lambda}$. The electron recoils at an angle φ

$$\mathcal{E} = \sqrt{p^2c^2 + m^2c^4}$$
$$\mathcal{E}_{\text{photon}} = pc$$
$$\mathcal{E}_{\text{rest}} = mc^2$$



a) (20 pts) Write the relativistic equations for momentum and energy conservation (using $\vec{p}, \widetilde{\vec{p}}, \mathcal{E}, \widetilde{\mathcal{E}}$ are the momenta and energies of the photon before and after the scattering, $\vec{p}_e, \widetilde{\vec{p}}_e, \mathcal{E}_e, \widetilde{\mathcal{E}}_e$, are the momenta and energies of the electron before and after the scattering)

$\vec{P}_e = 0$

momentum conservation

$$\vec{p} = \tilde{\vec{p}} + \tilde{\vec{P}}_e$$

energy conservation

$$\mathcal{E} + \mathcal{E}_e = \tilde{\mathcal{E}} + \tilde{\mathcal{E}}_e$$

for electron

$$\tilde{\mathcal{E}}_e = \sqrt{\tilde{P}_e^2 c^2 + m^2c^4} \qquad \mathcal{E}_e = mc^2$$

for photon

$$\tilde{\mathcal{E}} = \tilde{p}c \qquad \mathcal{E} = pc$$

$$\boxed{\begin{aligned} &\vec{p} - \tilde{\vec{p}} = \tilde{\vec{P}}_e \\ &pc + mc^2 = \tilde{p}c + \sqrt{\tilde{P}_e^2 c^2 + m^2c^4} \end{aligned}}$$

b) (20 pts) Find an expression for the change $\lambda - \widetilde{\lambda}$ in the photon wavelength for the special case $\theta = \pi/2$ (use $p = h/\lambda$)

① $\tilde{\vec{P}}_e^{\,2} = (\vec{p} - \tilde{\vec{p}})^2$ (from momentum conservation)

$(pc + mc^2 - \tilde{p}c)^2 = \tilde{P}_e^2 c^2 + m^2c^4$ (from energy conservation)

$\cancel{c^2}((p - \tilde{p}) + mc)^2 + \cancel{c^2}mc^2 = \tilde{P}_e^2 \cancel{c^2}$

② $\tilde{P}_e^2 = (p - \tilde{p})^2 + 2mc(p - \tilde{p})$

① $\rightarrow$ $\tilde{P}_e^2 = \tilde{\vec{P}}_e^{\,2} = \vec{p}\cdot\vec{p} + \tilde{\vec{p}}\cdot\tilde{\vec{p}} - 2\vec{p}\cdot\tilde{\vec{p}} = pp + \tilde{p}\tilde{p} - 2p\tilde{p}\cos\theta$

use ② $\longrightarrow$ $\cancel{pp} + \cancel{\tilde{p}\tilde{p}} - \cancel{2}p\tilde{p} + \cancel{2}mc(p - \tilde{p}) = \cancel{pp} + \cancel{\tilde{p}\tilde{p}} - \cancel{2}p\tilde{p}\cos\theta$

$p\tilde{p}(1 - \cos\theta) = mc(p - \tilde{p})$

$\theta = \pi/2 \qquad \cos\theta = 0$

$\frac{p\tilde{p}}{p\tilde{p}} = \frac{mc(p - \tilde{p})}{p\tilde{p}} \rightarrow 1 = mc\left(\frac{1}{\tilde{p}} - \frac{1}{p}\right)$

$h/\tilde{\lambda}$ h/λ

$$\boxed{\tilde{\lambda} - \lambda = \frac{h}{mc}}$$